Development and analysis of the large-span FRP woven web structure

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ABSTRACT: An innovative large-span structural system, namely the FRP woven web structure (FRPWWS), is introduced in this paper. In an FRPWWS, the high-strength FRP strips are "woven" like bamboo strips in a Chinese bamboo mat to form a plane web. The outer edge of the web is anchored on an outer ring beam, and an inner ring beam is provided to anchor the FRP strips at the center of the web. The stiffness of the web to resist various loads is derived from the initial prestressing during the "weaving" stage and the additional tensioning as a result of the out-of-plane movement of the inner ring beam. As a result of the high strength-to-weight ratio of FRP, this new structural form offers an attractive option for the construction of spatial structures with spans longer than are possible with conventional structural materials. In this paper, the basic layout and construction procedure for a simple FRPWWS is first presented. Three basic weaving patterns are next explained. Several variations of the basic structural system are also proposed. A simple mechanical model is presented for the deformation of individual FRP strips. Results from a finite element analysis of an example structure are also given. The results of these analyses confirm the feasibility of the FRPWWS.

1 INTRODUCTION

FRP is a new kind of structural material, whose use in civil engineering has been actively explored in recent years. Due to its favorable properties like corrosion resistance, high strength, low weight, good fatigue performance, and low maintenance cost, it is considered to be an ideal material for constructing long-span structures in the new century. However, its mechanical properties are distinctly different from those of traditional structural materials in some aspects, such as its anisotropy. Due to the unique properties of FRP, it is necessary to explore new forms of large-span structures for its efficient use and for achieving spans larger than are possible with traditional materials. For example, Maeda et al. (2002) have conceived a 5000 meter-span suspension bridge using FRP.

The FRP woven web structure, a new large-span structural system, is presented in this paper. This new system represents an attempt aimed at the efficient utilization of the unique characteristics of FRP in a large-span roof. In an FRPWWS, the high-strength FRP strips are woven like bamboo strips in a Chinese bamboo mat to form a plane web. The outer edge of the web is anchored on an outer ring beam, and an inner ring beam is provided to anchor the FRP strips at the center of the web. A small-scale model of a simple FRPWWS is shown in Figure 1. The FRP strips are initially prestressed to a limited extent to keep them straight during "weaving". Then, the FRP web is tensioned by a displacement of the inner ring beam in the out-of-plane direction, which is effected either by a set of prestressed tendons or by suspending a heavy mass from the inner ring beam. As a result, a tensioned FRP web, whose geometric stiffness is able to resist a variety of loads, forms a large-span roof system with the two rings.

The FRPWWS resembles the cable net structure and the cable-membrane structure: their members are flexible; and the geometric stiffness resulting from tension is utilized to resist loads. However, the FRPWWS has its unique advantages: (1) the FRP strips are ideal for super large-span structures due to their low self-weight and their superior material properties in the lengthwise direction, which are efficiently utilized, while the weakness of inferior properties in the transverse directions is not exposed;

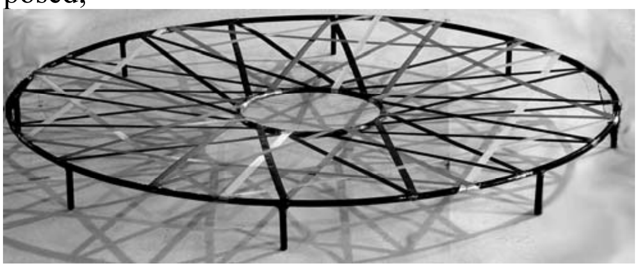


Figure 1. FRPWWS model.

(2) significant damping can be expected to arise from friction at joints between FRP strips, which can enhance the resistance of the structure to wind and earthquake loads; (3) the regular weaving pattern leads to an aesthetically pleasing surface; and (4) the corrosion resistance of FRP and the ease of installation because of its lightweight translate into low maintenance costs.

In this paper, the basic layout and construction procedure for a simple FRPWWS system is presented in detail. The weaving patterns in plane are summarized into three types. Some spatial FRPWWS forms for practical applications are also proposed. A simple mechanical model for individual FRP strips in the web is presented. Results from the finite element analysis of a simple FRPWWS are also described.

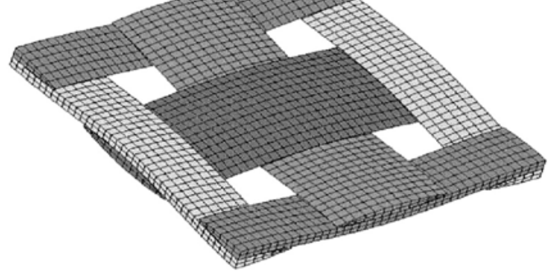
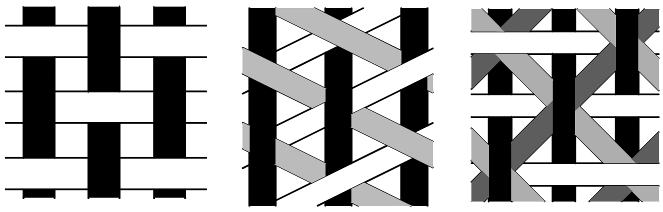


Figure 2. Part of a woven web (After Peng et al. 2004)



(a) (b) (c)

Figure 3. Weaving patterns

2 LAYOUT OF A SIMPLE FRPWWS

A simple FRP woven web structure is composed of a FRP woven web, an outer ring beam and an inner ring beam for anchorage, and an additional weight or a set of prestressed tendons, as shown in Figure 1.

The web is woven with FRP strips, and CFRP strips or other high-performance hybrid FRP strips are suggested. CFRP strips, which have been widely used to strengthen concrete structures in recent years, are manufactured by pultrusion in general, with a fiber volume ratio of about 65%. The properties of two representative products made in China and Switzerland respectively are listed in Table 1.

Table 1. Properties of two CFRP strip products.

Properties	Product 1	Product 2
Country of manufacturer	China	Switzerland
Width (mm)	100	120
Thickness (mm)	1.4	1.4
Specific gravity	1.5	1.6
Longitudinal strength (MPa)	≥2800	≥2400
Longitudinal modulus (GPa)	≥160	≥210
Ultimate elongation (%)	≥1.7	≥1.4
Thermal expansion coefficient (/℃)	0.2×10^{-6}	

The strips can be curved and circumvoluted due to their small thickness. A typical CFRP strip with properties similar to those shown in Table 1 is able to resist a tensile force of 400kN or more, while the weight of a 300m long strip is less than 70kg. In comparison, the self weight of a 300m long high strength steel cable which can resist the same load is more than 500kg.

The strips are arranged into a plane surface of a suitable pattern by some pre-defined rules. In the simplest weaving pattern, each strip passes over one crossing strip and under the next to form a web like a woven fabric. A part of such web is shown in Figure 2 (Peng et al. 2004). In a more general case, the number of strips meeting at a joint and the angles between these strips are the basic parameters of a weaving pattern: two strips at 90° to each other are shown in Fig. 3(a), three strips at 60° shown in Fig. 3(b), and four strips at 45° shown in Fig. 3(c). At the joints, strips can be fully inter-connected by adhesive bonding after complete shape formation or left unbonded so that sliding between strips is allowed. In the latter case, the static friction between strips can contribute to the stiffness under static loading while the sliding friction can consume the kinetic energy of the structure under dynamic loading.

3 CONSTRUCTION OF SIMPLE FRPWWS

Following the five construction steps as shown in Figure 4, a simplest FRPWWS can be completed.

First, the outer and inner ring beams on temporary supports are constructed. In general, the outer ring beam is in compression and the inner one is in tension when the web is in place. The outer ring beam is made of reinforced concrete while the inner ring beam is made of steel. The web which is woven with FRP strips is next fixed onto the ring beams with hinge joints and provided with some initial tension to form a plane surface. A tentative hinge joint scheme between the strip and the ring beam is shown in Figure 5, where the strip is tightly clamped between two stiff plates. The weaving of the strips should follow the rules of a specified weaving pattern which should have been designed by the structural engineer and the architect together. Weaving can be carried out easily due to the light weight and

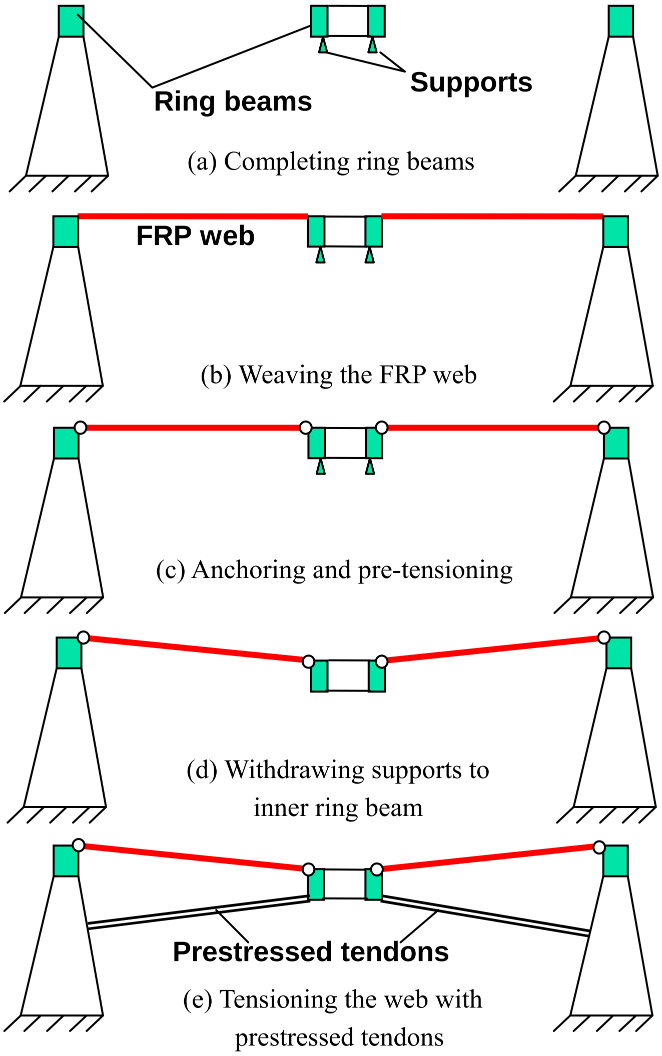


Figure 4. Construction steps

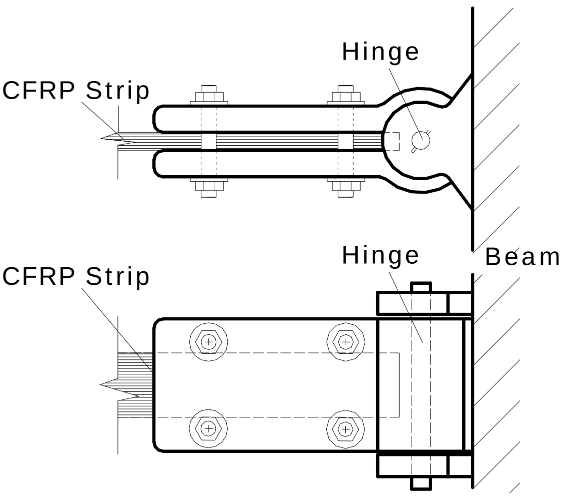


Figure 5. Tentative hinge joint scheme

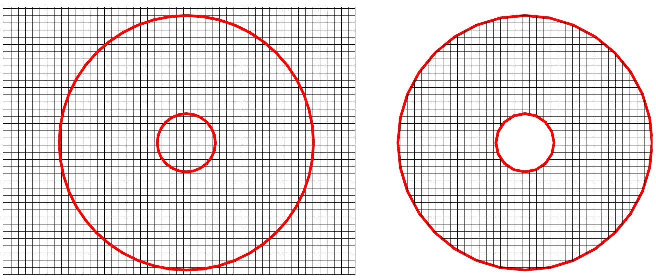


Figure 6. A tiled pattern with circular beams

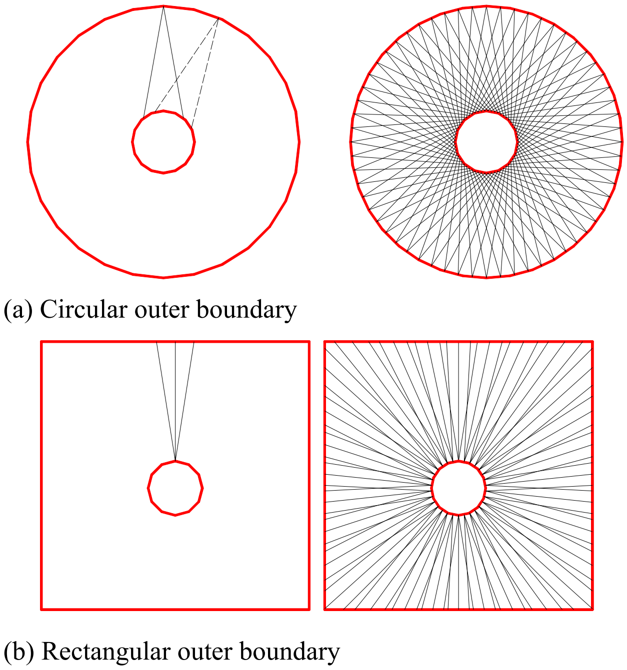


Figure 7. Radiated patterns

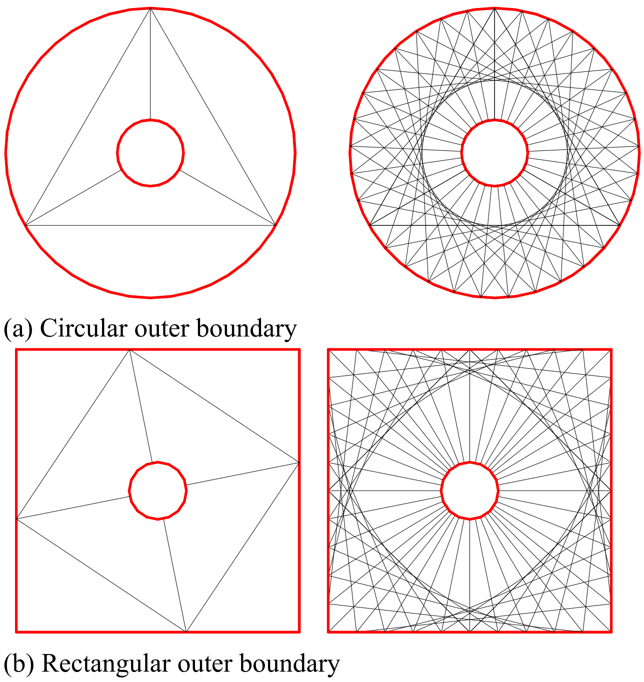


Figure 8. Polygonal patterns

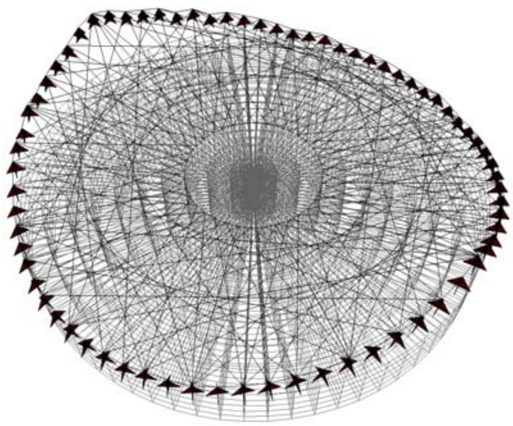


Figure 9. An FRPWWS with a spatially curved outer boundary

small thickness of FRP strips. The temporary supports to the inner ring beam are now withdrawn and the inner ring beam moves in the out-of-plane direction by its self-weight plus an additional weight where necessary. This out-of-plane displacement of the inner ring beam can also be effected by a set of high strength tendons. The installation of these prestressed tendons completes the construction process. According to force decomposition, a small out-of-plane force causes a large in-plane component and enables the web to be tensioned, leading to a web of sufficient stiffness to resist various loads.

4 MORE COMPLEX FORMS OF THE FRPWWS

4.1 *Basic plane weaving patterns*

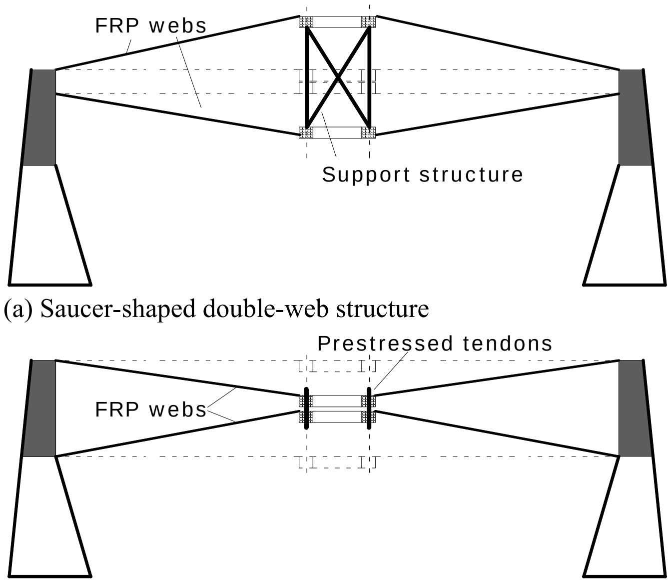
The FRP web is the main component in the FRPWWS system. Various weaving patterns can be adopted for the initial plane web, which will result in different mechanical behaviour. They may be classified into the following three types: tiled patterns, radiated patterns and polygonal patterns. A web weaving pattern that is independent of the boundaries is referred to as a tiled pattern as shown in Figure 6. The radiated pattern and the polygonal pattern are both made up of a number of repeated sets, each of which is composed of a number of line segments. Some examples of these two types are shown in Figures 7 and 8 respectively. Any pattern of these two types has its own defining rules. In practice, these three types of weaving patterns may be combined where appropriate in an FRPWWS system.

4.2 *Spatially curved outer ringbeam*

In a real structure, a spatially-curved outer ring beam may be adopted to achieve a more appealing building shape. If the outer edge of the FRP web is anchored onto such a ring beam, the strips can form a smooth curved surface in space, as shown in Figure 9. The shape is that of a piece of stressed cloth placed on the curved beam. Because the web is composed of individual strips, it offers great flexibility in forming a curved surface with good mechanical behavior.

4.3 *Double web system*

The load carrying capacity of an FRPWWS is mainly provided by its geometric stiffness derived from the tensile forces in the FRP strips. The strips are prestressed in two steps: initial prestressing in plane before anchorage to the ring beams and second stage prestressing through the out-of-plane movement of the inner ring beam. The former is applied to each strip one by one, to achieve a plane web surface and to control the total displacement. The latter is applied to shape the web and to achieve a predefined stress level, which can be realized in many different ways, including the use of prestressed tendons as mentioned earlier, uplifting with a stay column and a hung heavy weight on the inner ring beam, which may be retractable roof equipment. An alternative to the above approaches is to form two webs, whose inner ring beams are then pulled together or pushed apart to induce tensile forces in the strips. Such a double-web structure is illustrated in Figure 10. Figure 10(a) shows a saucer-shaped FRPWWS achieved by pushing apart the two inner ring beams, while Figure 10(b) shows a butterfly-shaped FRPWWS achieved by pulling together the two ring beams.



(a) Saucer-shaped double-web structure

(b) Butterfly-shaped double-web structure

Figure 10. Double-web systems

4.4 *FRPWWS with multiple ring beams*

There are only two ring beams in a simple FRPWWS. If more ring beams are used as shown in Figure 11, a large web will be divided into several shorter spans. The difficulty of construction and design will decrease as the length of continuous FRP strips becomes smaller. If a double-web system is provided with many ring beams, a folded-web system results (Figure 12).

The above are just some possible variations of the basic FRPWWS. In practical applications, many other forms/shapes can be explored. The FRPWWS may also be combined with other structural systems to become hybrid structural systems.

5 ANALYSIS OF A SIMPLE FRPWWS

5.1 *Individual FRP strip*

Each FRP strip in the web is mainly subject to tension if interaction between strips at joints is ignored.

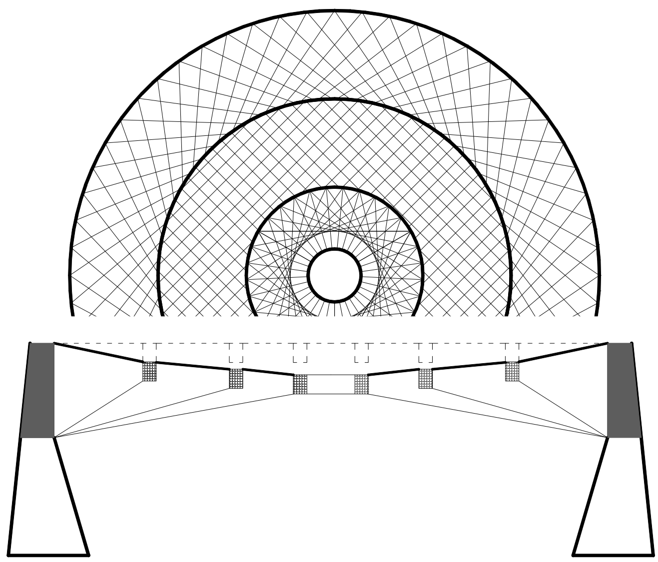


Figure 11. An FRPWWS with five ring beams

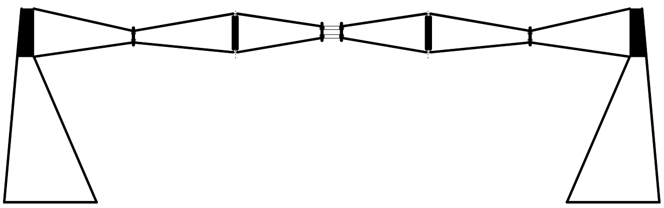
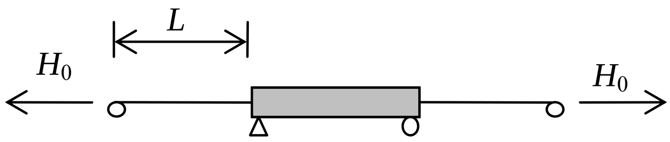
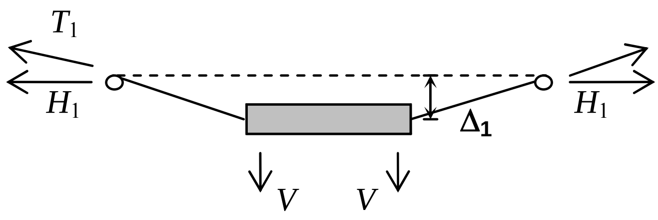


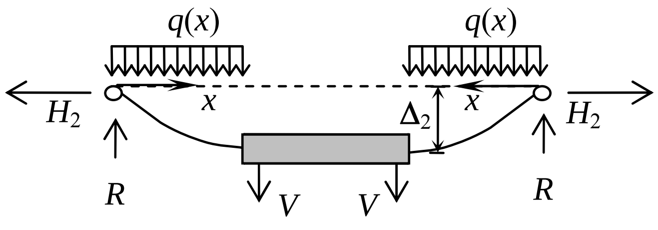
Figure 12. Folded FRPWWS



(a) Initial prestressed state



(b) Out-of-plane tensioned state



(c) Service loading state

Figure 13. Three states of a simple model of an FRPWWS

A pair of strips at 180° apart in a simple web can be modeled as two strips whose ends are connected to the outer beam and the inner ring beam respectively, as shown in Figure 13. The outer beam is regarded as a fixed point while the inner ring beam is treated as a rigid body. There are three states for the strips: initial prestressed state, out-of-plane tensioned state and service loading state.

It is assumed that the cross-sectional area of the strip is A, the elastic modulus is E, the difference between the radii of the inner and the outer ring beams is L, and the self weight and flexural stiffness of the FRP strip are neglected because they are very small.

In the first state as shown in Figure 13(a), a horizontal initial prestressing force H_0 is applied to the end of the strip. Then the strain of the strip is

$$\varepsilon_0 = \frac{H_0}{EA} \tag{1}$$

The initial length of the strip is

$$L_0 = \frac{L}{1+\varepsilon_0} \tag{2}$$

In the second state as shown in Figure 13(b), two vertical loads V are applied on the inner ring beam to move it down to tension the strips. The tensile force in the strip is T_1, whose horizontal component is H_1, and the displacement of the inner ring beam is Δ_1. Based on equilibrium consideration, there is

$$T_1 = \sqrt{{H_1}^2 + V^2} \tag{3}$$

If the strain of the strips at this time is ε_1, then

$$\varepsilon_1 = \frac{T_1}{EA} \tag{4}$$

$$\varepsilon_1 - \varepsilon_0 = \frac{\sqrt{L^2 + {\Delta_1}^2} - L}{L_0} \tag{5}$$

From the geometric relationship, there is

$$\frac{V}{H_1} = \frac{\Delta_1}{L} \tag{6}$$

Combining these equations, T_1, Δ_1, ε_1 can be found.

In the third state, the strips are required to support a service load $q(x)$ which is symmetrically placed with respect to the centre of the inner ring beam. The reaction at the fixed end can be decomposed into the horizontal force H_2 and the vertical force R. The total deflection of a point at a distance x from the fixed point is denoted by $z(x)$, the deflection due to the service load is denoted by $w(x)$, and the total displacement of the inner beam is denoted by Δ_2. Then,

$$R = \int_0^L q(x)\mathrm{d}x + V \tag{7}$$

The deflected shape is governed by the following cable equation (Shen 1997):

$$H_2 \frac{d^2 z}{dx^2} + q(x) = 0 \tag{8}$$

If $q(x)$ is a uniform load, the deflection curve can be found easily by double integration to be:

$$z(x) = \frac{q}{2H_2}(L - x)x + \frac{x}{L}\Delta_2 \tag{9}$$

and Equation (7) becomes

$$R = qL + V \tag{10}$$

The slopes at the strip ends are

$$\left.\frac{dz}{dx}\right|_{x=0} = \frac{R}{H_2} \tag{11}$$

$$\left.\frac{dz}{dx}\right|_{x=L} = \frac{V}{H_2} \tag{12}$$

Based on deformation compatibility, the total length change of each strip in the second stage is given by

$$\Delta s = \frac{1}{2}\int_0^L \left[\left(\frac{dz}{dx}\right)^2 - \left(\frac{\Delta_1}{L}\right)^2\right] dx \tag{13}$$

but the second stage elongation found from strains is

$$\Delta s = \frac{(H_2 - H_1)L}{EA}\left[1 + \left(\frac{\Delta_1}{L}\right)^2\right] \tag{14}$$

Thus, H_2 and Δ_2 can be found from Equations 10-14.

As an example, a pair of strips for a model FRPWWS as shown in Figure 13 is considered, where L=80m and the properties of the strips are the same as those of product 2 in Table 1. In the first stage, an initial stress of 500MPa is induced in the strips, then ε_0=0.0024, L_0=79.81m, and H_0=84kN. In the second stage, an out-of-plane force V=30kN is applied. As a result, Δ_1=7.14m, ε_1=0.0064, H_1=225.3kN and the stress in the strips is 1344MPa. Finally, a uniform load q=0.5kN/m is applied on the strips, and consequently Δ_2=10.27m, H_2=389.5kN, and the maximum stress in the strips is 2352MPa and occurs near the outer fixed end.

From this simple analysis, the key parameters for an FRPWWS can be identified. These include the initial control stress or the prestressing force H_0, the out-of-plane force V or the deflection of the inner beam Δ_1. They control the deformation of the web under loading and the stress level in the FRP strips.

5.2 *A Simple FRPWWS*

A simple FRPWWS as shown in Figure 14 was analyzed by the finite element method using the finite element package ANSYS (2000). It has a 150m span, and the radius of the inner ring is 30m. Product 2 listed in the Table 1 is employed in this structure, which is assumed to have a design value of 2000 MPa for the tensile strength. Only the radiated pattern is adopted. There are 360 repeated sets altogether, each of which is composed of three strips. Thus, there are 1,080 FRP strips in the structure. The geometric nonlinearity from large deformation was considered in the finite element analysis, while interaction between strips was neglected. The self weight of the FRP web is ignored as it is only 177kN which is much smaller the total load acting on the structure.

The stress level in the initial prestressed web is controlled to be no more than 210MPa, while that after the downward movement of the inner ring beam no more than 1000 MPa. From the finite element analysis, under a vertical force of 14,400 kN for the second stage operation, the inner ring beam moves down by 5.08 m and the maximum stress in the strips reaches 995MPa. The construction of the FRPWWS is now complete. As the weight of each strip in this FRPWWS is less than 18kg, which can be carried by an adult, the construction of this structure is expected to be easy. The total weight of the strips in the FRPWWS is estimated to be about 19,440 kg. The material cost of the FRP roof is about RMB 11 million, which is about 25% higher than the material cost of a steel cable net roof. However, the new system will incur lower construction and maintenance costs. As the span increases, this difference will decrease.

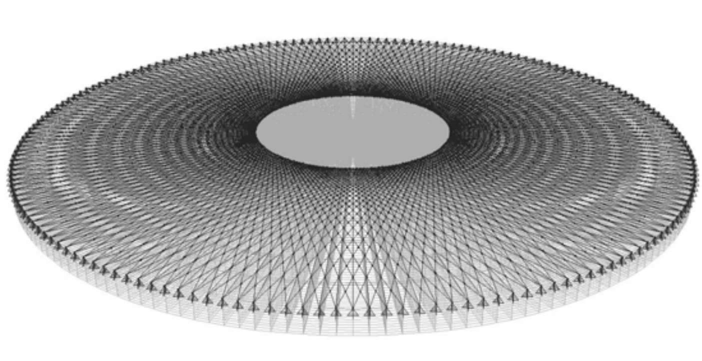


Figure 14. FRPWWS analysis model

The completed FRPWWS was next subject to a factored uniform load of 1.8kN/m^2 over the entire area enclosed by the outer ring beam, which is intended to include the self-weight of the roofing material, wind loading and snow loading. Under this loading, the maximum deflection increase is 1.22m and the total maximum stress in the FRP strips is 1542MPa, which is about 85.7% of the design stress. The structure is thus strong enough to resist this loading.

6 CONCLUSIONS

The FRP woven web structure (FRPWWS), which represents a new application of FRP in long-span structures, has been presented in this paper. The key aspects of this new system are listed below.

i. The FRP woven web, the ring beams and the out-of-plane tension system are the basic components of the FRPWWS.

ii. There are five construction steps for a simple FRPWWS as illustrated in Figure 4.

iii. The plane weaving patterns can be summarized into three types: tiled patterns, radiated patterns and polygonal patterns.

iv. Following the same basic principle, many different forms can be constructed. The paper has discussed several such possibilities, including the use of a spatially curved outer ring beam, the double-web system and systems with multiple ring beams.

v. An FRPWWS experiences three distinct stress states: the initial prestressed state, the out-of-plane tensioned state and the service loading state, all of which should be considered in design.

7 ACKNOWLEDGEMENTS

The authors are grateful to the Natural Science Foundation of China for their support to the research presented here through a national key project on the application of FRP composites in civil engineering in China (Project No. 50238030) and through the Joint Research Fund for Hong Kong and Macao Young Scholars (Project No. 50329802).

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