MECHANICAL BEHAVIOUR OF NEW FRP/ALC BUCKLING-RESTRAINED SANDWICH SLAB SYSTEM

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Abstract
An alternative new sandwich slab system consisting of ultra-lightweight autoclaved lightweight aerated concrete core and high rigid and strength carbon FRP surface skins is proposed in this study. The general criterion for the elastic local buckling of thin-walled skins on elastic foundation is theoretically solved and the local buckling-restraint condition is clearly proposed as a simple equation. It has been shown that the sandwich slabs consisting of an ALC core adhesively bonded with various FRP skins satisfy this buckling-restraint condition. The effects of adhesive rigidity on the deflection of FRP/ALC sandwich slabs have been discussed.

Keywords: autoclaved lightweight aerated concrete, sandwich panel, slab, FRP skin.

1. Introduction
FRP sandwich panel structures which have relatively low stiffness urethane or phenol form core may collapse due to the occurrence of elastic local buckling of the skin [1]. Then the merit of the use of relatively expensive FRP skin plate will decrease. On the other hand, the authors previously performed the four-pointed bending tests of the steel skin sandwich slab having ALC (autoclaved lightweight aerated concrete) core [2] as shown in Figure 1 and discussed that the bonded steel skins on the surface of the ALC core show no elastic local-buckling and the shear fracture of ALC core determines the strength of the sandwich slab system. They also in [3] pointed out that this kind of sandwich slab system has the mechanical benefit of significant reduction of bending deflection and the dramatic increases of load-carrying capacities compared to no-reinforced ALC slabs [4].
Considering both ultra-lightning and anti-corrosion, the sandwich slabs consisting of an ALC core adhesively bonded with FRP skins are adopted in this paper. First of all, the mechanism of the buckling restrain due to the effect of core stiffness is clarified through the energy method analysis for simple modelled structural stability problem. Then the effects of various stiffness of adhesive bond material on the deflection will be discussed.

2. Elastic local buckling analysis for the skin of sandwich panels

For a thin-walled skin plate on an elastic foundation having width \( b_f \) and wall-thickness \( t_f \) as shown in Figure 2, the buckling condition associated with the buckling mode using variational principle [5,6] may be given as

\[
\delta (U_{2h} + V_{2m} + U_{2c}) = 0
\]  

(1)

where the first, second and third terms are respectively the quadratic component of the bending strain energy of the skin plate, the quadratic part of the interactions between the prebuckling state and the nonlinear membrane stresses and strains, and the quadratic component of the linear membrane strain energy of the core.

An alternative simplified two-dimensional buckling analysis in which the analytical region is limited and concentrated on only a typical lobe of the incremental deformed length \( l \) and core depth \( h \) as shown in Figure 2. This treatment is very similar to that which has been already developed in [1]. Then the buckling displacement of the skin plate \( w \) associated with the vertical direction \( z \) and that of the core \( w_c \) are respectively written as
\[ w = \alpha \sin \frac{\pi x}{l}, \quad w_c = \alpha \frac{z}{h} \sin \frac{\pi x}{l} \]  

(2)

where \( \alpha \) is the unknown amplitude of the buckling mode.

Using Eq. (2), the quadratic energy components of Eq. (1) are written as

\[ U_{2b} = \frac{b_l}{2} \int_0^l \frac{E_f (t_f)^3}{12} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \, dx = \alpha^2 E_f \frac{l b_l (t_f)^3}{48} \left( \frac{\pi}{l} \right)^4 \]  

(3)

\[ V_{2m} = \frac{b_l}{2} \int_0^l (-\sigma_f) \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \, dx = -\alpha^2 \sigma \frac{\pi^2 b_l t_f}{4 l} \]  

(4)

\[ U_{2c} = \frac{b_l}{2} \int_0^l c w^2 \, dx = \alpha^2 c \frac{l b_l}{4} \]  

(5)

where \( E_f \) is the Young’s modulus of skin plate, \( \sigma \) the prebuckling compressive force per area in longitudinal direction and \( c \) the equivalent rigidity of the elastic core-foundation. Now Eq. (5) would consist of the vertical \( z \)-directional membrane strain energy \( U_{2A} \) and the shear strain energy \( U_{2S} \) written as

\[ U_{2A} = \frac{b_l}{2} \int_0^l \int_0^h E_c \left( \frac{\partial w_c}{\partial z} \right)^2 \, dx \, dz = \alpha^2 E_c \frac{l b_l}{4 h} \]  

(6)

\[ U_{2S} = \frac{b_l}{2} \int_0^l \int_0^h 4G_c \left( \frac{1}{2} \frac{\partial w_c}{\partial x} \right)^2 \, dx \, dz = \alpha^2 G_c \frac{\pi^2 b_l h}{12 l} \]  

(7)

where \( E_c \) and \( G_c \) are respectively the Young’s modulus and the shear modulus of the core. Under the condition \( U_{2c} = U_{2A} + U_{2S} \), the following relation is obtained.

\[ c = \frac{E_c}{h} + \frac{\pi^2 G_c}{3 l^2} h \]  

(8)

The effectiveness depth of the core as the buckling restraint foundation \( h_e \) may be calculated through \( \partial c / \partial h = 0 \) for Eq. (8) then

\[ h_e = \frac{l}{\pi} \sqrt{\frac{3 E_c}{G_c}} \]  

(9)

By substituting Eqs (3), (4) and (5) into Eq. (1), the eigenvalue criterion is obtained using \( l = b_f t_f^3 / 12 \) and \( A = b_f t_f \) as

\[ \sigma_{cr} = \frac{\pi^2 E_f l}{l^2 A} + \frac{t_f^2 E_c}{\pi^2 t_f h_e} + \frac{h_e G_c}{3 t_f} \]  

(10)

The first term of Eq. (10) means the so-called Euler buckling stress for the simply supported plate-column. The second relates to the effect of spring stiffness of elastic foundation, and the third term is due to the shear rigidity of the core. Using the minimisation of Eq. (10) on
account of buckling wave length $l$, the buckling wave length associated with the minimum buckling stress $\sigma^*$ is obtained as

$$l^* = \pi f \left( \frac{\left( E_f \right)^2}{48 E_c G_c} \right)^{1/6}$$

(11)

Then the local buckling stress as the minimum buckling stress associates with $l^*$ is

$$\sigma^* = \frac{\pi^2 E_f I}{(l^*)^2 A} + \frac{(l^*)^2 E_c}{\pi^2 t_f h^*} + \frac{h^* G_c}{3 t_f}$$

for $h^* = \frac{l^*}{\pi} \sqrt{\frac{3E_c}{G_c}}$

(12)

3. Young’s modulus of core for restraining the skin from local buckling

It is easy understood from Eqs (11) and (12) that as the core rigidity increases, the local buckling stress of the skin $\sigma^*$ increases and the buckling length $l^*$ as well as the associated restraint zone of core $h^*$ decrease. If $\sigma^*$ is larger than the material yielding or failure stress $F_f$, any local buckling effect does not need to be considered in design, that is, the following condition may be expected,

$$\sigma^* = \frac{\pi^2 E_f I}{(l^*)^2 A} + \frac{(l^*)^2 E_c}{\pi^2 t_f h^*} + \frac{h^* G_c}{3 t_f} \geq F_f$$

(13)

When an isotropic material (like ALC) having the Poisson’s ratio $\nu_c$ is used as core, through $G_c = E_c / \left[ 2(1 + \nu_c) \right]$ and Eq. (11), the simple formulation for the necessity of local buckling restraint core rigidity $E_{cr}$ is written as

$$\left( \frac{E_{cr}}{E_f} \right)^2 \geq \frac{8(1 + \nu_c)}{3} \left( \frac{F_f}{E_f} \right)^3$$

(14)

If $\nu_c \approx 0$, we obtain as

$$\left( \frac{E_{cr}}{E_f} \right)^2 \geq \frac{8}{3} \left( \frac{F_f}{E_f} \right)^3$$

(15)

Using Eq. (14) or (15), it would be easy recognised that in the present FRP/ALC sandwich slab, the local buckling of skin is negligible.

4. Effects of adhesive rigidity on the deflection of FRP/ALC slabs

The commercially available finite element software, NX Nastran, has been used; the solid finite elements with eight-nodes are adopted for the two-types of model as shown in Figure 3. The ALC core has a span $L$ and a width $b_c$, FRP surface skins are of $t_f = 1.2$ mm thickness, and the adhesive bond layers have Young’s modulus $E_a$ and thickness $t_a = 1.0$ mm. Shown in Figure 4 for $b_f = b_c$ are that the center deflection increases remarkably when the ratio of
$E_a$ to Young’s modulus of steel $E_s (= 205 \text{kN/mm}^2)$ becomes to be less than $10^{-4}$. Four solid curves in Figure 4 show the present FEA results for the Young’s modulus of the skins $E_f = 30 \text{kN/mm}^2$, $50 \text{kN/mm}^2$ and $100 \text{kN/mm}^2$ for FRPs and $205 \text{kN/mm}^2$ for steel. The dotted curves are the results based upon the well-known composite beam theory considering bending and shear deflections as,

$$\delta_c = \frac{23PL^3}{1296E_f I_e} + \frac{PL}{5A_c G_c}$$

where $I_e$ is the effective moment of inertia for the composite slab beam and $A_c$ the cross sectional area of ALC core. The centre deflection value in Figure 4 is normalised by that of only ALC plate.

![Figure 3. Loading condition and analytical models.](image)

![Figure 4. Centre deflections in the case of $b_f = b_c$.](image)
\[ \delta_0 = \frac{23PL^3}{108E_c(t_c)^3b_c} + \frac{PL}{5A_cG_c} \]  

(17)

In these examples, the ratios of \( E_d / E_s \) for mortal, epoxy and silicon bonds are around \( 10^{-1} \), \( 10^{-2} \) and \( 5 \times 10^{-6} \), respectively. Shown in these figures are that the composite beam theory in solid lines is available to estimate deflection behaviours for \( E_d / E_s \geq 10^{-3} \) which is satisfied in various normally used epoxy bonds. But when a very soft silicon bond is adopted, the composite beam theory yields dangerous estimations; the shear deflection in the bond layers should be considered and an alternative FEA may be recommended.

5. Conclusions

The general criterion for the elastic local buckling of thin-walled skins on elastic foundation is theoretically solved and the local buckling-restraint condition has been proposed as a simple equation. Then the sandwich slabs consisting of an ALC core adhesively bonded with FRP skins satisfy this buckling-restraint condition. It has been also shown that the classical simple composite beam theory is available to estimate the deflection behaviours for the Young’s modulus ratio \( E_d / E_s \geq 10^{-3} \) which is satisfied in normally used epoxy bonds.

References


