THE EFFECT OF FRP STIFFNESS VARIATION ON FRP DEBONDING FROM CONCRETE

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Abstract
In wet lay-up fiber reinforced polymer (FRP) bonded concrete members, fiber sheets are applied by hand and the cured FRPs do not have constant properties. Therefore, the behaviour of FRP bonded concrete specimens varies among specimens even the same materials are used. In this research, a mathematical method is used to predict the variation of load versus deflection responses of FRP bonded concrete specimens subjected to Mode I and Mode II loading due to the variation of FRP properties. In an effort to effectively model the variation of FRP properties, a random white noise using a one-dimensional standard Brownian motion is added to the governing equations of FRP debonding from concrete subjected to Mode I and Mode II loading, yielding a stochastic differential equation. The bounds of load vs. deflection responses with 95% probability are found for different experimental tests. The variations of FRP stiffness for these tests are also determined by a systematic method.

Keywords: Brownian motion, concrete, FRP stiffness, Mode I, Mode II, white noise.

1. Introduction
Extensive research in previous two decades has clearly shown that externally bonded fiber reinforced polymer (FRP) composites have good potential for use in strengthening of concrete members [1,2]. A common method to use FRP to strengthen concrete is the wet lay-up bonding, that consists of installation by hand using unidirectional dry fiber sheets or fabrics impregnated with a saturating resin on-site. The stiffness (bending stiffness and tension stiffness, hereafter
referred to as “stiffness” only unless the tension or bending stiffness needs to be emphasized) of FRP can affect significantly the responses and behaviour of strengthened members. However, it is very hard to obtain the accurate magnitude of the stiffness of FRP applied by the wet lay-up process. Generally manufacturers provide the fibre sheets properties such as Young’s modulus and design sheet thickness based on the tests in laboratories. However, if the wet lay-up process is used to apply FRP onto concrete, the stiffness of cured FRP is affected by the skills of the workers who apply the FRP, and also by the curing process in field. Typically it is hard to get the same level of quality control for wet lay-up FRP as for the FRP strips precured in factory. Therefore, the wet lay-up operation may cause significant differences between what is in the manufacturer’s reports and the actual FRP stiffness achieved in-situ. In addition, the FRP properties may be different at different locations even in the same specimen because the fibres may be curved to different extents at different locations. Since the FRP stiffness plays an important role in predicting the load-carrying capacity and failure modes in FRP retrofitted or strengthened concrete members, it is useful to find a way to determine the range of FRP stiffness variations in the members without testing each of them.

Brownian motion is a mathematical model used to describe random movements like random drifting of particles suspended in a fluid. Mathematically, a Brownian motion is a continuous stationary stochastic process, \( W(t) \), having independent increments, and for each \( t, W(t) \) is a Gaussian random variable with mean value of 0 and variance \( t \). White noise can be considered as the derivative of a Brownian motion existing in the stationary sense [3]. Generally, white noise is a random signal that can be applied to model a totally unpredictable process.

In this study, a random white noise is added to the FRP stiffness parameter in the governing equations to represent the differences between the theoretical FRP stiffness calculated from manufacturer’s reported properties and the actual values. The governing equations for FRP debonding from concrete subjected to Mode I and Mode II loading become stochastic differential equations, where the driven noise is a one-dimensional standard Brownian motion. By solving these equations and comparing the results with the experimental data, the ranges of FRP stiffness with 95% probability are determined for the experimental tests found in literature.

2. FRP-to-concrete interface under mode II loading condition

Figure 1 shows an FRP-to-concrete interface under a pull-out action, i.e. the specimen is subjected to Mode II loading. In this figure, \( a_0 \) is the initial crack length between the FRP sheet/plate and the concrete substrate and \( \Delta \) is the shear displacement between them at the loaded point of the FRP. There is a linear relationship between the in-plane ultimate shear loading, which is the axial force, \( N_{max} \), in the FRP sheet, and the maximum shear displacement, \( \Delta_{max} \) [1]:

\[
N_{max} = \left( \frac{E_f t_f b_f}{a_{max}} \right) \Delta_{max}
\]

where \( E_f \) = elastic modulus of FRP; \( t_f \) = thickness of FRP; and \( b_f \) = width of FRP and \( a_{max} \) = maximum crack length. Eq. (1) can be rewritten as:

\[
\frac{dN_{max}}{d\Delta_{max}} = K_0, \quad K_0 = \frac{E_f t_f b_f}{a_{max}}
\]

As discussed in the previous section, the actual FRP tension stiffness achieved in-situ is not the same as this calculated value; it is inevitably subject to random fluctuations, resulting from the wet lay-up process and different amount of concrete attached to the debonded FRP at different locations.
locations. In order to find the effect of FRP tension stiffness variation on the ultimate load-carrying capacity (i.e. the maximum pull-out load) of the specimens, a white noise is added to the parameter $K_0$ as in Eq. (3).

$$K = K_0 + \alpha \dot{w}$$

(3)

where $\alpha$ is a constant value that presents the distribution of variation of experimental data, and $\dot{w}$ is a one dimensional white noise which is a function of the property variation due to the construction process and different amount of concrete attached to the debonded FRP at different locations, and all of them affect the measured value of $\Delta$. Therefore, $\dot{w}$ is a function of $\Delta$. By substituting Eq. (3) into Eq. (2), the axial force can be written as:

$$dN_{max} = K_0 d\Delta_{max} + \alpha \dot{w} d\Delta_{max}$$

(4)

Since white noise can be formally regarded as the derivative of a Brownian motion [3], Eq. (4) can be rewritten as:

$$dN_{max} = K_0 d\Delta_{max} + \alpha w(\Delta_{max}),$$

(5)

where $w(\Delta_{max})$ is a one-dimensional standard Brownian motion. Recall that [3] a stochastic process, $w$ is a Brownian motion having independent and stationary increments, and for each $\Delta_{max}$, $w(\Delta_{max})$ is a Gaussian random variable with mean 0 and variance $\Delta_{max}$. In this application, a Brownian motion is introduced to present the effect of totally unpredictable FRP stiffness shifts from its theoretical value on the load vs. deflection responses of the specimens. Eq. (5) is a stochastic differential equation and the solution is [5]:

$$N_{max} = K_0 \Delta_{max} + \alpha w(\Delta_{max})$$

(6)

In the probability theory, if an arbitrary random variable $X$ has Gaussian distribution with mean $\mu$ and variance $\sigma^2$, then $\frac{X - \mu}{\sigma}$ has a standard normal distribution with mean 0 and variance 1 [3].

As mentioned before, Brownian motion $w(\Delta_{max})$ has Gaussian distribution with mean 0 and variance $\Delta_{max}$. Therefore, $\frac{w(\Delta_{max})}{\sqrt{\Delta_{max}}}$ has a standard normal distribution. According to the probability density function, 95% of the observations fall between -1.96 and 1.96 for a variable with the standard normal distribution as shown in Figure 2. So with probability of 95%,
\[-1.96 < \frac{\omega(\Delta_{\text{max}})}{\sqrt{\Delta_{\text{max}}}} < 1.96\]  

By substituting Eq. (6) into (7), the upper and lower bounds can be found for the maximum axial force, \(N_{\text{max}}\):

\[-1.96 < \frac{\mathcal{N}_{\text{max}} - K_0 \Delta_{\text{max}}}{\alpha \sqrt{\Delta_{\text{max}}}} < 1.96\]

\[
\begin{cases}
N_{\text{lower}} = K_0 \Delta_{\text{max}} - 1.96 \alpha \sqrt{\Delta_{\text{max}}} \\
N_{\text{upper}} = K_0 \Delta_{\text{max}} + 1.96 \alpha \sqrt{\Delta_{\text{max}}} 
\end{cases}
\]

In order to use data from different tests with different materials, \(K_0 \mathcal{N}_{\text{max}}\) against \(K_0 \sqrt{\Delta_{\text{max}}\,\text{curves}}\) are drawn in Figure 3. In this figure, the thin line represents the theoretical results calculated from Eq. (1). The thick lines are the results from Eq. (8) and * points are the experimental data from the literature [6]. By adjusting the value of \(\alpha\) in Eq. (8), the band between the lower and upper \(N\) values can be changed to involve all experimental data. In this study, \(\alpha = 8\) seems to be a good estimation to cover all experimental data from [6].

![Figure 3. Application of experimental data to find \(\alpha\)](image)

Regarding to \(\mathcal{N}_{\text{max}}\) is between \(N_{\text{upper}}\) , \(N_{\text{lower}}\) and \(\varepsilon_{\text{max}} = \frac{\Delta_{\text{max}}}{\alpha_{\text{max}}}\), the bounds of FRP tension stiffness accompanied with noise, \(E_f t_f\), with probability 95% are:

\[
\left( E_f t_f - \frac{1.96 \alpha \sqrt{\Delta_{\text{max}}}}{b_f \varepsilon_{\text{fmax}}} \right) < E_f t_f < \left( E_f t_f + \frac{1.96 \alpha \sqrt{\Delta_{\text{max}}}}{b_f \varepsilon_{\text{fmax}}} \right)
\]

where \(\varepsilon_{\text{fmax}}\) = maximum FRP strain at the loaded end. For the specimen CR1L3 in [6], the properties are: \(E_f t_f = 75.90 \text{ kN/mm} \), \(b_f = 100 \text{ mm} \), \(\varepsilon_{\text{fmax}} = 0.00496\), and \(\Delta_{\text{max}} = 0.307 \text{ mm}\) [6]. With \(\alpha = 8\) which is determined in previous discussion, the tension stiffness for CR1L3 is in the range of 58.39 to 93.41 KN/mm. If the inequality (9) is applied for all examined experimental data used in Figure 3 in order to determine their tension stiffness boundaries, the
real stiffness falls between 31% less or more than the theory stiffness, so \((E_t t)_\text{real} = (E_t t)_\text{theory} \pm 31\%\).

3. FRP-to-concrete interface under mode I loading condition

The relation between load and displacement in a Mode I loading condition is more complicated compared to a Mode II loading case. Figure 4 shows the schematic model around the crack tip subjected to a Mode I load, \(P\). There is a crack (debonded) length, \(a\), existing between the FRP and the concrete. If the value of \(a\) is small, the debonded FRP can be assumed as a beam subjected to bending force, and the bonded parts of FRP can be assumed as an Euler-Bernoulli beam on an elastic Winkler foundation [7]. Therefore, the relationship between the displacement, \(\Delta\), and the Mode I load, \(P\), may be obtained as follows [8]:

\[
P = K_0 \Delta, \quad K_0 = \left(\frac{6E_t \beta^3}{2\alpha^3 \beta^3 - 6\alpha \beta^2 + 3}\right), \quad \beta = \frac{4k_e}{AE_t}
\]

where \(E_t\) = the bending stiffness of FRP; \(k_e\) = the stiffness factor of springs connecting the FRP and the concrete and is taken as \(E_a b_f / t_a\) [8], in which \(E_a\) = the elastic modulus of bonding adhesive; \(b_f\) = the width of FRP; and \(t_a\) = the thickness of bonding adhesive.

To model the unpredictable changes in FRP bending stiffness, \(E_t\), a white noise is added to the parameter \(K_0\), where \(K_0\) is calculated based on reported FRP material properties. The process is similar to what is done for the Mode II loading case. The upper and lower bounds for Mode I loading, \(P\), with probability 95% are:

\[
\begin{align*}
P_{\text{lower}} &= K_0 \Delta - 1.96 \alpha \sqrt{\Delta} \\
P_{\text{upper}} &= K_0 \Delta + 1.96 \alpha \sqrt{\Delta}
\end{align*}
\]

To find the magnitude of \(\alpha\), experimental data from [9] have been used as shown in Figure 5. By using the experimental data, the appropriate magnitude for \(\alpha\) is equal to 4 for the data in [9]. Figure 6 presents the load against displacement for one of the samples in [9], C4B1S1. The properties for this sample are \(E_f = 155000\) MPa, \(t_f = 1.9\) mm, \(b_f = 76\) mm, \(\Delta_{\text{max}} = 0.2\) mm, and \(K_e = 801.04\) MPa. When \(\Delta\) is approaching to \(\Delta_{\text{max}}\), the bounds of load obtained from Eq. (11) approach to a linear relation with \(\Delta\) as shown in the figure. Therefore, a linear relation is assumed between \(P_{\text{lower}}\) and \(P_{\text{upper}}\) and \(\Delta\). The slopes of these assumed linear bounds, shown with thin solid straight lines in Figure 6, are the maximum and minimum values
of the parameter $K$. Therefore,

$$
\begin{align*}
\rho_{\text{max}} &= \left[ K_0 - \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] \Delta \\
\rho_{\text{min}} &= \left[ K_0 + \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] \Delta
\end{align*}
$$

Therefore,

$$
\left[ K_0 - \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] < K < \left[ K_0 + \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right], \quad K = \left[ \frac{6E\beta^3}{2\alpha^3\beta^2 - 6\alpha\beta^2 + 3} \right]
$$

Figure 5. Application of experimental data to find $\alpha$

Figure 6. Load against FRP strain for C4B1S1 in [9].

The inequality (13) cannot be directly used to find the bounds or range for the actual FRP bending stiffness. By substituting $K = \left[ \frac{6E\beta^3}{2\alpha^3\beta^2 - 6\alpha\beta^2 + 3} \right]$ and $\beta = \frac{4}{\sqrt{4E\ell}}$ into the inequality (13), it can be rewritten as follows:

$$
\begin{align*}
Y_1 &= \left[ K_0 - \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] * \left( \frac{a^2}{3} - \frac{1.414}{\sqrt{k_e}} aEI^{0.25} + \frac{1.414}{4\sqrt{k_e}} * EI^{0.75} \right) - EI < 0 \\
Y_2 &= \left[ K_0 + \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] * \left( \frac{a^2}{3} - \frac{1.414}{\sqrt{k_e}} aEI^{0.25} + \frac{1.414}{4\sqrt{k_e}} * EI^{0.75} \right) - EI > 0
\end{align*}
$$

Y1 and Y2 are fourth degree functions with variable $EI^{0.25}$ that have just one acceptable answer ($EI^{0.25} > 0$) to satisfy inequalities (14). For $EI^{0.25} > 0$, Y1 and Y2 are descending functions. Therefore, the answer of $Y1=0$ is the minimum $EI^{0.25}$ that causes $Y1<0$ and answer of $Y2=0$ is the maximum $EI^{0.25}$ that causes $Y2>0$. Therefore it can be concluded:

$$
\begin{align*}
EI + \left[ K_0 - \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] * \left( \frac{a^2}{3} - \frac{1.414}{\sqrt{k_e}} aEI^{0.25} + \frac{1.414}{4\sqrt{k_e}} * EI^{0.75} \right) &= 0 \text{ for solving } EI_{\text{min}} (15) \\
EI + \left[ K_0 + \frac{1.96\alpha}{\sqrt{\beta_{\text{max}}}} \right] * \left( \frac{a^2}{3} - \frac{1.414}{\sqrt{k_e}} aEI^{0.25} + \frac{1.414}{4\sqrt{k_e}} * EI^{0.75} \right) &= 0 \text{ for solving } EI_{\text{max}}
\end{align*}
$$
To illustrate the application of Eq. (15), the functions \( Y1 \) and \( Y2 \) for the sample C4B1S1 are plotted in Figure 7. It can be seen in the figure that \( Y1 < 0 \) when \( EI > 4445000 \text{N/mm}^2 \) and \( Y2 > 0 \) when \( EI < 9060000 \text{N/mm}^2 \). Therefore, the bounds of FRP bending stiffness for this sample are equal to 4445000 and 9060000 N/mm², respectively, while the real bending stiffness is equal to 6733300 N/mm². The big range of FRP bending stiffness is mainly from the big difference of concrete amount attached to debonded FRP at different locations in this specimen.

If the Equation (15) is applied for experimental data used in Figure 5 to find their bending stiffness boundaries, the actual stiffness falls approximately between 36% less and more than the theory stiffness, so \( (E_fI_f)_{\text{real}} = (E_fI_f)_{\text{theory}} \pm 36\% \).

4. Conclusion

In this study, white noise and Brownian motion concepts in probability theory are used to model the variation of FRP stiffness in FRP bonded concrete specimens subjected to Mode I and Mode II loading. A systematic method is established to determine the range of FRP stiffness in FRP bonded concrete specimens. Experimental data from literatures are used to demonstrate the validity of this method. For the experimental data used in this research, tension stiffness varies 31% away from its theoretical value while the bending stiffness varies away 36%.

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6. References


