STUDIES ON FRP-CONCRETE INTERFACE WITH EXPONENTIAL OR LINEAR SOFTENING BOND-SLIP LAW

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Abstract

The latest test study proposes a theory that the bond-slip law for a FRP-concrete interface contains linear hardening and exponential softening. On the basis of this law, the paper studies the mechanic behavior and deboning process of a FRP-concrete interface. Firstly, through nonlinear fracture mechanics, the analytical solutions of the interface shear stress, the axial normal stress of FRP and the load-displacement relationship at the loaded end with the single shear test model of FRP-concrete are acquired. The shear stress propagation as well as the debonding process of the whole interface for different bond lengths could be predicted. Secondly, a simplified interface bond-slip law is used by changing the exponential softening law into a linear softening law. In addition, the analytical solutions for the simplified interface bond-slip law could also be obtained. Finally, based on the analytical solutions of the two bond-slip laws, the influences of the FRP bond length and stiffness on load-displacement curve and the ultimate load are discussed.

Keywords: effective bond length, FRP, hardening, load-displacement relationship, softening, ultimate load

1. Introduction

model for the single side FRP reinforced concrete model to find the analytical solution of FRP-concrete interface. Analytical solutions in closed-form for the complete debonding process are also available for a local bond–slip law with linear softening [14] and nonlinear softening[15].

Based on the latest test results of FRP-concrete interface with linear hardening and exponential softening bond-slip constitutive [16], this paper studies the FRP-concrete interface debonding process. Closed-form solutions are given.

2. Interface model of FRP-to-concrete

2.1 Interface Model

The adhesive bonded joint analyzed, shown in Figure 1, may be considered as a simple and typical model of FRP-strengthened structures in order to understand the stress transfer and debonding behavior. The adhesive layer is mainly subjected to shear (mode II fracture). The thickness and width of the two layers are $t_p$ and $b_p$ for the upper steel/FRP laminate, and $t_c$ and $b_c$ for the lower concrete plate, respectively. The Young’s moduli of the steel/FRP laminate and the concrete plate are $E_p$ and $E_c$, respectively. $L$ is the bonding length. Before starting the derivations, the following assumptions can be made for simplicity of problems:

- The adherents are homogeneous and linear elastic;
- The adhesive is only exposed to shear forces;
- Bending effects are neglected;
- The normal stresses are uniformly distributed over the cross-section; and
- The thickness and width of the adherents are constant throughout the bond line.

![Figure 1. Adhesive bonded joint](image)

2.2 Governing Equations

Considering the element shown in Figure 2, the equations of equilibrium for adherents can be written as

$$\frac{d\sigma_p}{dx} - \frac{\tau}{t_p} = 0 \tag{1}$$

$$\sigma_p t_p b_p + \sigma_c t_c b_c = 0 \tag{2}$$

where $\tau$ is the shear stress in the adhesive layer. The constitutive equations for the adhesive
layer and the two adherents are expressed as

$$\tau = f(\delta)$$  \hspace{1cm} (3)

$$\sigma_p = E_p \frac{du_p}{dx}$$  \hspace{1cm} (4)

$$\sigma_c = E_c \frac{du_c}{dx}$$  \hspace{1cm} (5)

where \(u_p\) and \(u_c\) are the displacements of FRP and concrete. The interface slip \(\delta\) is defined as the relative displacement of two bonded layers.

$$\delta = u_p - u_c$$  \hspace{1cm} (6)

Substituting (2)-(5) into (1) yields, by introducing two parameters of local bond strength \(\tau_f\) and interfacial fracture energy \(G_f\)

$$\frac{d^2 \delta}{dx^2} + \frac{2G_f}{\tau_f^2} \lambda^2 f(\delta) = 0$$  \hspace{1cm} (7)

$$\sigma_p = \frac{\tau_f}{2G_f} \frac{d\delta}{dx}$$  \hspace{1cm} (8)

where

$$\lambda^2 = \frac{\tau_f^2}{2G_f} \left( \frac{1}{E_p t_p} + \frac{b_p}{b_c E_c t_c} \right)$$  \hspace{1cm} (9)

Eq. (7) is the governing differential equation of the adhesive bonded joint in Figure 1. When the local bond-slip law was given, the equation can be solved.

### 2.3 Bond-slip Law

Recent study shows the local bond-slip law for some kind of bonded joints can be described as Figure 3. The interfacial shear stress increase linearly to \(\beta \tau_f\) (\(\beta<1\)) when the corresponding interface slip increase to \(\alpha \delta_1\) (\(\alpha<1\)). It is called as elastic state (state I). Then the interfacial shear stress increase linearly to peak \(\tau_f\), while the interface slip increase from \(\alpha \delta_1\) to \(\delta_1\). It is called as hardening state (state II). When the interface slip attains to \(\delta_1\), the interface softening appears, and the interfacial shear stress decays exponentially with the interface slip.
It is called as softening state (state III). The mathematics expressions of the interface bond-slip law in Figure 3 are

\[
f(\delta) = \begin{cases} 
\frac{\beta}{\alpha \delta_i} \delta, & 0 \leq \delta \leq \alpha \delta_i \\
\frac{\tau_f}{1-\alpha} \left[\frac{(1-\beta)\delta}{\delta_i} + (\beta - \alpha)\right], & \alpha \delta_i < \delta \leq \delta_i \\
-\frac{\tau_f}{\delta_f - \delta_i} (\delta - \delta_i), & \delta < \delta_i \leq \delta_f \\
0, & \delta > \delta_f 
\end{cases}
\]  

(10)

This bond-slip law is called exponential model in this paper. \(G_f\), the interfacial fracture energy of the bond-slip law is the area enclosed by the curve and the \(\delta\) axis in Figure 3.

\[
G_f = G_{fI} + G_{fII} + G_{fIII} = \frac{1}{2} (1 - \alpha + \beta) \tau_f \delta_i + k 
\]  

(11)

where \(G_{fI}\), \(G_{fII}\) and \(G_{fIII}\) are the areas under the curves, associated with the elastic, hardening and softening region, respectively. Based on the above bond-slip law, the interfacial shear stress, axial normal stress in FRP and the load-displacement relationship at loaded end could be obtained.

In Figure 4, the exponential softening model is simplified. When the interface slip increases to \(\delta_i\), the softening appears, then the interface shear stress decreases linearly with the slip. When the interface slip attains to \(\delta_f\), the interfacial shear stress reduces to zero. It is called as debonding state (state IV). The mathematics expressions of the interface bond-slip law in Figure 4 are

\[
f(\delta) = \begin{cases} 
\frac{\beta}{\alpha \delta_i} \delta, & 0 \leq \delta \leq \alpha \delta_i \\
\frac{\tau_f}{1-\alpha} \left[\frac{(1-\beta)\delta}{\delta_i} + (\beta - \alpha)\right], & \alpha \delta_i < \delta \leq \delta_i \\
-\frac{\tau_f}{\delta_f - \delta_i} (\delta - \delta_i), & \delta_i < \delta \leq \delta_f \\
0, & \delta > \delta_f 
\end{cases}
\]  

(12)
The above bond-slip law is called tri-linear model in the paper. And the expression $\delta_f$ could be got by letting interfacial fracture energy $G_f$ in Figure 3 and 4 equal.

$$\delta_f = \frac{2k}{\tau_f} + \delta_1$$  \hspace{1cm} (13)

Based on the tri-linear model, the interfacial shear stress, axial normal stress in FRP and the load-displacement relationship at loaded end could also be got.

3. Analysis of the debonding process for exponential model

3.1 Long Bond Length $L > h_0$

With the bond–slip model defined in Figure 3, the governing equation (Eq. (7)) can be solved to find the shear stress distribution along the interface and the load–displacement response of the bonded joint. The solution can be presented below stage by stage with illustrations of the corresponding interfacial shear stress distribution and load–displacement curves. If the bond length $L$ is longer than $h_0$, the failure process would experience elastic stage, elastic-hardening stage, elastic-hardening-softening stage, and softening stage.

3.2 Short Bond Length $L < h_0$

The bond length $L$ is longer than $h_0$ in the previous section. If the bond length $L$ is shorter than $h_0$, as the slip equals to $a_0$ at $x=0$, the slip does not reach to $\delta_1$ at $x=L$, yet. The failure process would only experience elastic stage, elastic-hardening stage, hardening stage, hardening-softening stage, and softening stage. The critical bond length $h_0$ can be obtained as in Eq. (14)

$$\cosh(\lambda_2 h_0) = 1/\beta$$ \hspace{1cm} (14)

where

$$\lambda_2^2 = \frac{2G_f}{\tau_f^2} \lambda_1^2 \cdot \frac{1 - \beta \cdot \tau_f}{1 - \alpha \cdot \delta_1} = \frac{1 - \beta \cdot \tau_f}{1 - \alpha \cdot \delta_1} \cdot \left( \frac{1}{E_p t_p} + \frac{b_p}{b_e E_c t_c} \right)$$ \hspace{1cm} (15)

4. Analysis of the debonding process for tri-linear model

Three values of critical bond length $L_0$, $a_0$, $h_0$ are identified with different failure processes for the tri-linear model. These values are given as follows

$$L_0 = h_0 + a_0$$ \hspace{1cm} (16)

$$a = a_u = \frac{\pi}{2 \lambda_3}$$ \hspace{1cm} (17)

Where $h_0$ is as shown in Eq. (14), and

$$\tan(\lambda_3 a_0) = \frac{(\delta_f - \delta_1) \lambda_3 \sqrt{1 - \beta^2}}{(1 - \alpha)(1 + \beta) \lambda_3 \delta_1}$$ \hspace{1cm} (18)

$$\lambda_3^2 = \frac{2G_f}{\tau_f^2} \lambda_1^2 \cdot \frac{\tau_f}{\delta_f - \delta_1} = \frac{\tau_f}{\delta_f - \delta_1} \cdot \left( \frac{1}{E_p t_p} + \frac{b_p}{b_e E_c t_c} \right)$$ \hspace{1cm} (19)

4.1 Long Bond Length $L > L_0 = h_0 + a_0$

The failure process would experience elastic stage, elastic-hardening stage, elastic-hardening-softening stage, elastic-hardening-softening-debonding stage, hardening-softening-debonding stage, and softening-debonding stage.
4.2 Bond Length $a_u<L<L_0$

The failure process would experience elastic stage, elastic-hardening stage, elastic-hardening-softening stage, hardening-softening stage, hardening-softening-debonding stage, and softening-debonding stage.

4.3 Bond Length $h_0<L<a_u$

The failure process would experience elastic stage, elastic-hardening stage, elastic-hardening-softening stage, hardening-softening stage, and softening stage.

4.4 Short Bond Length $L<h_0$

The failure process would only experience elastic stage, elastic-hardening stage, hardening stage, hardening-softening stage, softening stage.

5. Numerical simulations

The material properties and geometry parameters in numerical analysis are selected as follows: $t_p=1.4\text{mm}$, $b_p=30\text{mm}$, $t_c=200\text{mm}$, $b_c=200\text{mm}$, $E_p=152.2\text{Gpa}$, $E_c=32.5\text{Gpa}$. And the interfacial characteristic parameters are selected as $\alpha=0.5$, $\beta=0.7$, $k=0.8\text{N/mm}$, $\delta_1=0.07\text{mm}$, $\tau_f=5.6\text{Mpa}$.

From Eqs (13)-(14) and (16)-(18), it could be obtained that $\delta_f=0.3557\text{mm}$, $h_0=59.5\text{mm}$, $a_0=118.9\text{mm}$, $a_u=163.4\text{mm}$, $l_0=h_0+a_0=178.4\text{mm}$. According to above analysis, the failure process for different bond lengths could be discussed.
Figure 5. Comparison to load-displacement curves of the two models for different bond lengths

Figure 5 shows the comparison of load-displacement curves between the exponential model and tri-linear model for different bond lengths. \( \Delta \) stands for the slip at loaded end \( x=L \). It can be concluded that the ultimate load would be larger in tri-linear model as the intact softening region may appears on the interface. The load-displacement curves would be different when softening area exists. And as there has no debonding situation in exponential model, the displacement at the ended load would increase unlimited in the softening stage, that is different from the tri-linear model in which the slip would finally approach to \( \delta_f \).

6. Conclusions

Closed form solutions are obtained for both linear hardening-exponential softening and trilinear interfacial constitutive model. The shear stress propagation as well as the debonding process of the whole interface for different bond lengths could be predicted.

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