NUMERICAL MODELING OF EXISTING RC BEAMS STRENGTHENED IN SHEAR WITH FRP U-SHEETS

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Abstract
This paper presents a finite element analysis on two reinforced concrete beams, extracted from an important building of the ’30s in Rome, strengthened in shear with C-FRP and subjected to the simultaneous occurrence of a negative bending moment and shear stresses. The commercial program ATENA was used to perform numerical simulations of the tests. The numerical results seem to be in good agreement with the experimental data in terms of load–displacement response, strain state, crack pattern and failure mode.

Keywords: Existing building, Shear strengthening, CFRP, FEM analysis.

1. Introduction
The FRP materials have been introduced mainly to retrofit RC structures damaged or unable to sustain service or seismic loads. Up to date, a significant amount of experimental research works has been carried out in order to investigate the behavior of RC elements reinforced with FRP as well as to define the design guidelines. However, most of these studies are referred to simply supported beams (beam ends strengthening in positive moment regions) and therefore some design approaches may be inaccurate to satisfy the real stress conditions affecting the structure (i.e. frame structures) with strengthening also in negative moment regions (beam ends near the nodes). In order to accurately determine stress and strain...
distributions both in concrete and FRP as well as to provide realistic predictions of retrofitted beams, numerical procedures have been pointed out. Although many finite element analyses on plated RC beams retrofitted with FRP plates have been carried out, little success has been achieved in simulating brittle failures [1]. Perera et al. [2] proposed a nonlinear numerical model based on damage mechanics, that permits consideration of the position and increase of concrete cracks that have a very important influence on the behavior of strengthened beams. The model also simulates the stress distribution and the failure mechanism. Lopez [3] developed an approach by numerical model that can reproduce the global load–deflection response of RC beams strengthened with C-FRP laminates and the changes in strain along the length of the FRP laminates. Buyle-Bodin et al. [4] proposed a finite element model to analyze and predict the flexural behavior of strengthened and repaired beams.

In this paper, the use of a finite element method by the program ATENA [5] has been adopted to model the behavior of beams strengthened with C-FRP. The experimental results of two strengthened RC beams tested at the Experimental Laboratory of the Structural Department of the University “Roma Tre” to investigate their shear behavior in case of simultaneous occurrence of negative bending moment and shear stresses [6], are used for validation of the numerical models.

2. Beams models

The numerical analyses are performed to simulate the experimental campaign, conducted at the Experimental Laboratory of the Structural Department of the University “Roma Tre” on two RC beams (marked as TM1 and TM2) extracted from a building in Rome dated the mid-’30s ([6]-[7]). The two beams are characterized by a T-section, smooth longitudinal rebars and stirrups with spacing varying from 21 to 38 cm. In order to generate a negative moment on the support, a cantilever is added to one end of the original beams.

The retrofitting of the beams consists in the application of C-FRP strips for the strengthening in shear and the construction of a r.c. slab for the strengthening in flexure.

Two static tests have been performed on the repaired beam TM1: a first test (TM1a) aimed to simulate the effects due to construction defects and a second one (TM1b) without construction defects. The beam TM2 was strengthened as the beam TM1b. A more accurate description of the experimental investigation is reported in Imperatore et al. (2012) [6]. Some numerical analyses were performed to model the nonlinear behavior of the tested beams by the FEM software ATENA [5]: to simulate the beams TM1b and TM2 without defects, (Figure 1a) three-dimensional models are built; to simulate the beam TM1a with defects, a plane model is used (Figure 1b). In this case, as experimentally found out, the beam behavior is governed by the construction defects, the stress state is very modest for the C-FRP strips and the C-FRP bond law can be neglected.

The two-dimensional model consists of several macroelements with a 2 x 2 cm square mesh, to reproduce the different beam parts with different thickness and/or materials. In particular, four macroelements were created for the web parts (two for the zone reinforced with C-FRP, one for the span and one for the cantilever) and other four for the respective flanges. The new slab was modelled with three macroelements (to simulate the slab, the cantilever flanges and the cantilever web, respectively). The experimental load steps are applied to the model and the solution was calculated using the Newton-Raphson algorithm.

The three-dimensional models consist of: a macroelement for the existing beam, two macroelements for the cantilever (one to add above the slab and one for the load application) and six macroelements for the new slab. The mesh is composed of tetrahedral of 10 cm side. The experimental load steps are applied to the model and the solution is calculated using the
Newton-Raphson algorithm as long as possible while the Arc-Length method is applied in the case of convergence problems.

The applied loading steps simulate the experimental shear/bending moment ratio. For a better comparison with the experimental results, the control points were introduced at the same positions of the experimental measurement points.

Figure 1. a) 2D Finite element model of the beam TM1a in ATENA 2D; b) 3D Finite element model of beams TM1b and TM2 in ATENA 3D.

3. Materials model

3.1 Concrete model

The concrete model used for the numerical investigation is the SBeta Constitutive model [5], that is characterized by:

- non-linear behavior in compression including hardening and softening,
- fracture of concrete in tension based on the nonlinear fracture mechanics,
- biaxial strength failure criterion,
- reduction of compressive strength after cracking,
- tension stiffening effect,
- reduction of the shear stiffness after cracking,
rotated crack model.

More specifically, the non-linear fracture mechanics and a crack band method that utilizes the smeared crack concept are combined in the tensile concrete behavior: the cracks in concrete occur when major principal stress exceeds the tensile strength; after the crack initiation (controlled by a bi-axial failure envelope), the isotropic material formulation changes to an orthotropic one. The stress on softening curve is determined by the crack opening displacement, calculated from the inelastic cracking strains.

The mechanical properties of the new concrete (used for the cantilever and the slab) are evaluated by means the conventional EC2 formulation [8], on the basis of the experimental compressive strength. In particular, a C28/35 concrete is used for the cantilever and for the slabs of the beams TM2 and TM1b. The slab of the beam TM1a, instead, is built with a lightweight concrete with polypropylene fibers (40 x 12 x 0.2 mm) characterized by a mean cubic compressive strength of 61.70 MPa and an elastic modulus of 31.62 GPa.

The existing beam TM1 is characterized by a medium concrete cubic strength of 19.1 MPa evaluated by destructive tests on specimens extracted by the concrete core. The beam TM2 has a concrete strength of 25.5 MPa evaluated on the basis of no-destructive SONREB tests. Concrete elastic modulus and tensile strength are calculated by Model Code 90 [9] and reduced to account the cracking. The reduced mechanical properties for the concrete of the beams TM1 and TM2 are shown in table 1 and are obtained using a trial and error approach based on the evaluation of the degradation of the tensile strength and elastic modulus in experimental cyclic tests.

| Table 1. Mechanical properties of existing concrete for beams TM1 and TM2 |
|-----------------|-----------------|
| TM1             | TM2             |
| Elastic Modulus | 4194 MPa        | 4613 MPa        |
| Tensile strength| 0.75 MPa        | 0.85 MPa        |
| Compressive strength | 16.20 MPa     | 20.88 MPa       |
| Fracture energy | 0.16 N/m        | 0.19 N/m        |

Finally, in order to simulate the contact stress of the construction joints between the beam and the new slab and between the beam and the added cantilever an interface material model is used. This material model is based on the Mohr-Coulomb criterion (Figure 2); in the original un-deformed geometry, the interface lines have identical location and there is full interaction of the contact sides; after, a cut-off tension a residual surface dry friction forms.

![Figure 2. Interface model for the construction joints (i.e. between the beam and the slab); (a)Failure criterion, (b) Normal stiffness, (c) Tangential stiffness.](image)

3.2 Steel and C-FRP reinforcement models

Steel rebars and C-FRP strips may be modelled as a discrete reinforcement by truss elements or as smeared reinforcement (through a steel reinforcement ratio associated to the concrete section; this solution is particularly suitable to model the stirrups). In the present study, the smeared reinforcement is limited to model the shear reinforcement and the FRP into the beam TM1a and the cantilever shear reinforcement into the beams TM2 and TM1b. In all other cases, the discrete reinforcement is utilized.
An elasto-plastic law with hardening and the Von Mises yield criterion describe the behaviour of the main reinforcement. As experimentally found, the longitudinal rebars of the existing beam are characterized by a yield strength of 265.82 MPa and an ultimate strength of 404.87 MPa whereas the stirrups mechanical properties are a yield strength of 351.95 MPa and an ultimate strength of 479.89 MPa.

The cantilever reinforcement consists in steel rebars type B450C. The slab reinforcement consists in a ø6/20 electro-welded net (ultimate strength of 587.49 MPa) and layers of longitudinal rebars (ultimate strength of 594.55 MPa). The first is modelled as a smeared reinforcement, the second as discrete one. The U-shape rebars introduced to connect the slab to the beams TM2 and TM1b are also modelled by means of discrete elements.

Bond laws are associated to the reinforcement elements: a perfect connection for ribbed rebars of the new reinforcement (Figure 1a) and the bond law proposed by Bigaj et al. [10] for the existent reinforcement are chosen. The stress-slip model depends on the bond quality, concrete cubic compressive strength and reinforcement bar radius and is characterized by a curve that is parabolic until the bond strength is reached, then bi-linear until failure (Figure 3).

<table>
<thead>
<tr>
<th>Bond quality</th>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
<th>Point 4</th>
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<td>0.040</td>
<td>0.047</td>
<td>0.480</td>
</tr>
<tr>
<td></td>
<td>0.500</td>
<td>1.000</td>
<td>0.700</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Adimensional parameters for defining the bond stress-slip relationship for smooth bars

Figure 3. Bond law for smooth steel.

The model for C-FRP strips exhibits only linear elastic behaviour up to failure. The C-FRP has a tensile strength of 538.74 MPa, elastic modulus of 65.5 GPa, as measured.

A smeared reinforcement with perfect bond is used for 2D model (beam TM1a) due to the lack of important delamination phenomena during the test.

A discrete reinforcement simulates the C-FRP in the 3D models (beams TM1b and TM2). In this case, the bond law defined in the CNR-DT 200/2004 [11] guidelines is utilized.

4. Numerical results of the beam models

The numerical results on the beam TM1a (2D model) present some differences respect to the experimental ones. This discrepancy can be attributed to the ineffectiveness of the model in capture the effects of local phenomena emerged in the experimental test due to the construction defects. The comparison between the experimental and the analytical results on the beams TM1b and TM2 (3D models) shows a good fit in terms of vertical deflection (Figure 4).

Figure 5 shows the deformed configuration of the two-dimensional model for the beam TM1a at the collapse with its crack pattern obtained from the numerical simulation: the model manages to capture the detachment of the slab and the formation of the shear cracks at the top of the original beam. Low stress values for the C-FRP strips according to the experimental results, were observed by recording the tensile principal strains of the model. The model can catch also the crushing of concrete at the right support B.

The analytical results have highlighted the influence of construction details such as the absence of the connectors in the center of the beam on the response of the reinforced beam. At the moment the beam stiffness is not well reproduced due to the difficult definition of the existent beam material properties including the state of cracking.
Figure 4. Force-displacement curves to 0.6 times the length of the span

Figure 5. a) principal tension strain-deformation state and cracks at failure; b) cracks opening (m). Detail of the zone where the slab lifted.

The beam TM1b experimentally presented local phenomena due to buckling of the lower existent beam reinforcement near the support B (Figure 6). Due to this reason, differences arise comparing the C-FRP principal strains with the experimental results (Figure 7, top). The beam TM2, instead has not presented local phenomena during the experimental test and the numerical results show a good agreement with experimental ones (Figure 7, bottom).
5. Conclusions

In order to comprehend the behaviour of two reinforced RC beams extracted from a '30s RC structure, finite element models are built with ATENA. The beams are retrofitted in shear with C-FRP strips and strengthened in flexure with a new additional RC slab along the extrados of the beam.

In particular a 2D model, characterized by a minor computational effort, simulates the behaviour of the retrofitted beam with some construction defects during the retrofitting intervention (TM1a). In this case, a FRP bond law is not necessary as the C-FRP strips are not stressed during the experimental test because of a premature failure due to local phenomena associated to the construction defects.

3D models, which include accurate bond slip model for C-FRP and concrete, are used for the beams retrofitted correctly (TM1b, TM2) because of the C-FRP worked during the tests.

Proper material models are used for C-FRP strips, existing concrete, new concrete, steel rebars and construction joints interfaces.

Different model solutions are used for C-FRP strips: a smeared model for 2D models and a discrete model with truss elements adding a bond law for 3D models.

Appropriate bond laws are used for different type of steel reinforcement: perfect bond for new ribbed rebars and Bigaj’s model for the existing rebars.

It has been observed that proposed beam models reproduce well the experimental results and collapse in absence of local phenomena.

The analytical results have highlighted the influence of construction details such as the absence of the connectors in the center of the beam on the response of the reinforced beam.

Further efforts should be made to improve the numerical results in terms of beam stiffness.
6. Acknowledgements

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7. References


