FRACTURE MECHANICS APPROACH TO NONLINEAR DEBONDING PROBLEMS IN RC BEAMS STRENGTHENED WITH COMPOSITE MATERIALS

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ABSTRACT

The nonlinear debonding mechanism in reinforced concrete beams strengthened with externally bonded FRP is analytically investigated. The paper focuses on beams with pre-existing debonded regions and on cases in which various loading scenarios yield compressive stresses in the bonded strip. Under these conditions, the FRP strip tends to buckle, which, in turn, may trigger the unstable growth of the debonded region. The high order geometrically nonlinear model of the strengthened beam is combined with the fracture mechanics concept of the energy release rate to provide a quantitative criterion for the initiation, propagation, and stability of the debonding process. Numerical results that demonstrate the capabilities of the proposed modeling approach and throw light on some of the physical phenomena associated with the structural response of the strengthened beam are presented. A summary and conclusions closes the paper.

KEYWORDS

Debonding, FRP, RC beams, Strengthening, Analytical Modelling, Energy Release Rate.

INTRODUCTION

Strengthening, upgrading, and retrofitting of existing structures by means of externally bonded fiber reinforced polymer (FRP) composite materials have gained worldwide acceptance in the past two decades (Bakis et al (2002)). However, along with its many advantages, this method is characterized by the formation of brittle modes of failure, and especially the development of unstable and sudden debonding failure modes (Buyukozturk and Hearing (1998)). These modes include debonding that initiates near the edge of the composite material strip (the "cut-off" section) and rapidly propagates towards mid-span and the unstable burst of pre-existing debonded regions. Such regions may result from flexural and flexure-shear cracking of the concrete beam, from improper placement and fastening of the bonded strip, or from localized damage to the bonding layer. In all cases, the debonding failure is governed by the stress concentrations near the cut-off section (Buyukozturk et al (2004), Lau et al (2001), Smith and Teng (2002a, 2002b), Rabinovitch and Frostig (2000), Teng et al (2002)) or near unbonded regions throughout the beam (Rabinovitch and Frostig (2001)).

The debonding mechanism in general, and especially the sudden and unstable growth of pre-existing un-bonded regions, is further accelerated due to the formation of compressive stresses in the thin composite strip. Although the strengthening system is usually designed to carry tensile forces and its use to resist compressive forces is not recommended (ACI committee 440.2R-02 (2002)), in some cases, the exposure of the FRP laminate to compressive stresses is inevitable (see Rabinovitch (2004a)). Among these cases, one can list moment and curvature reversal under different loading cases; strengthening to resist cyclic and dynamic loads; combined axial and flexural loading; lateral loading of frame members; unloading of members strengthened while carrying loads; strengthening on two faces of the element; and others.

While FRP materials can support compressive stresses, microbuckling of fibers and buckling or wrinkling of the laminate can occur (ACI committee 440.2R-02 (2002), Uy et al (2002)). Furthermore, the combination of the buckling of the FRP strip with the tendency of the debonded region to propagate has a critical influence on the functionality of the strengthening system and on the safety of the strengthened beam.

In this paper, the challenge of quantitatively describing the nonlinear behavior, buckling, and growth of the debonded region through a general modeling approach is addressed. To achieve this goal, two fundamental analytical methodologies are employed. The first one uses the high order analytical model of the strengthened
RC beam (Rabinovitch and Frostig (2000)) along with the consideration of the geometrical nonlinearity of the various constituents (Rabinovitch 2004a). The second analytical methodology that is employed here uses the fracture mechanics concept of the energy release rate as the criterion for the development and stable or unstable growth of the debonded region. The fracture mechanics criterion is preferred over the standard stress criteria (Varastehpour and Hamelin (1997), Malek et al (1998), Mukhopadhyaya and Swamy (2001)) due to the brittle character of the debonding crack.

The paper presents the mathematical model for the geometrically nonlinear analysis of the strengthened beam with emphasis on the distinction between the fully bonded and the debonded regions. Since the influence of the development of compressive forces in the bonded laminate on the buckling and growth of debonded regions within the strengthened beam (inner debonding) is the most prominent, the paper focuses on this problem. The mathematical modeling approach and the energy release rate criterion are presented in the mathematical formulation section. A numerical study that highlights the geometrically nonlinear behavior of a strengthened beam with an inner debonding and quantitatively examines the tendency of the debonded region to grow is also presented. A summary and conclusions closes the paper.

MATHEMATICAL MODEL

The mathematical formulation presents the geometrically nonlinear analytical model for the strengthened beam. In this model, a distinction is made between strengthened and fully bonded regions and strengthened but debonded regions. The analytical model uses the large displacements and moderate rotations kinematic relations for the mathematical representation of the RC beam and the FRP strip. The adhesive layer is modeled as a 2D elastic medium that resists shear and vertical normal stresses, while its inplane rigidity is neglected (see Rabinovitch and Frostig (2000)). The closed form stress and displacement fields of the adhesive are based on 2D linear elasticity whereas the beam and lamination theories are employed for the RC beam and the FRP strip, respectively. In the fully bonded regions, it is assumed that the adhesive-concrete and adhesive-FRP interfaces resist shear and vertical normal stresses. On the other hand, in the debonded regions, it is assumed that the debonded FRP strip is fully detached and the adhesive layer is free of shear and vertical normal stresses. Finally, it is assumed that the stress and deformation fields are uniform through the width of the beam.

The notation, sign convention, and coordinate systems used in the mathematical formulation appear in Figure 1.

\[ \begin{align*}
\text{Figure 1: Notation and sign conventions (a) deflections and coordinates (b) loads (c) stresses and stress resultants}
\end{align*} \]

The notation, sign convention, and coordinate systems used in the mathematical formulation appear in Figure 1. The governing equations of the fully bonded region are stated in terms of the unknown vertical deflections in the vicinity of the debonded region. The nonlinear governing equations take the following form (Rabinovitch (2004a)):

\[ \begin{align*}
&EA''w''_x + EA'w'_xw'_x + b\tau'' = -n''_x \\
&A_{il}''u''_{x,c} + A_{il}''w''_xw''_x + b\tau'' = -n''_o \\
&-EF''w''_x + EA''w''_xw'_x + \frac{3}{2}EA''(w'_x)^2 w''_x + EA''u''_xw'_x + b\left(\frac{c_a}{2} + \gamma\right)\tau'_z + \frac{hE}{c_a}(w''_x - w''_o) = -q''_x \\
&-D_{il}''w''_x + A_{il}''w''_xw''_x + \frac{3}{2}A_{il}''(w'_x)^2 w''_x + A_{il}''u''_xw''_x + b\left(\frac{c_a + d''_o}{2}\right)\tau'_z - \frac{hE}{c_a}(w''_x - w''_o) = -q''_o \\
&u''_x - u''_o + \frac{c_a}{G_c}\tau'' - \frac{c_a}{12E_c}\tau''_z = \left(\frac{c_a}{2} + \gamma\right)w''_x - \left(\frac{c_a + d''_o}{2}\right)w''_o = 0
\end{align*} \]

where the superscripts "rc", "a", and "frp" refer to the RC beam, the adhesive material, and the FRP strip, respectively; \( EA'' \) and \( EF'' \) are the inplane and flexural rigidities of the RC beam, \( A_{il}'' \) and \( D_{il}'' \) are the inplane
and flexural rigidities of the FRP strip, respectively, $E_a$ and $G_a$ are the elastic and shear moduli of the adhesive material, respectively; $q'$ and $n'$ ($i=rc, frp$) are vertical and horizontal distributed loads, respectively; $Y^x$ is the height of the centroid axis in the RC beam; $c_a$ and $d^p$ are the heights of the adhesive layer and the FRP strip, respectively; $b$ is the width of the FRP strip and the adhesive layer; and $f_j$, denotes derivative with respect to $x$.

In the debonded regions, the shear and vertical normal stresses in the adhesive layer vanish and the governing equations reduce to

$$\begin{align}
E_a^\prime w^\prime_{,xx} + E^\prime a w^\prime_{,xx} &= -n^\prime \\
A^\prime a \left( u^\prime_{,xx} + A^\prime \right) w^\prime_{,xx} &= -n^p
\end{align}$$

$$\begin{align}
- El_{xx} w^\prime_{,xx} + E'a w^\prime_{,xx} + \frac{3}{2} Ea' (w^\prime_{,xx})^2 w_{,xx} + E'a' w_{,xx} w_{,xx} &= -q^\prime \\
- D^p_{xx} w^p_{,xx} + A^p \left( u^p_{,xx} + A^p \right) w^p_{,xx} &= -q^p
\end{align}$$

Equations (1-5) and (6-9) form two sets of nonlinear ordinary differential equations that govern the structural behavior of the fully bonded and the debonded regions, respectively. Due to the nonlinearity of the model, the governing equations do not have a general closed form solution. Alternatively, they are solved using a nonlinear multiple shooting algorithm (Stoer and Bulirsch (1993)). Also, note that in the debonded region, the shear stresses in the adhesive layer vanish, hence it is no longer an unknown. Correspondingly, the order of the equations reduces from 14 in the bonded region to 12 in the debonded one. This effect requires a unique set of continuity conditions at the connection point between the two adjacent regions (at $x=x_j$). In physical terms, the connection point between the fully bonded and the debonded regions represents the tip of the interfacial debonding crack. In the RC beam ($i=rc$) and the FRP strip ($i=frp$), these conditions require continuity of the deformations and equilibrium of the internal stress resultants and the external loads as follows:

$$\begin{align}
u^{(i)} &= u^{(i)}, & w^{(i)} = w^{(i)} \\
N^{(i)} = N^{(i)} \\
M^{(i)} + M^{(i)} &= M^f
\end{align}$$

where ($i$) refers to quantities left and right of the connection point $x=x_j$; $z_{im}$ denotes external concentrated loads and bending moments exerted at the connection point. In the adhesive layer, the connectivity conditions require that the shear stress in the fully bonded region at $x=x_j$ equals to zero.

The criterion for the propagation of the interfacial crack and the growth of the debonded region adopts the fracture mechanics concept of the energy release rate (Brock (1974), Rabinovich and Frostig (2001)). Following this concept, the propagation of the horizontal debonding crack occurs if $G$, the amount of the energy released by the structure due to an infinitesimal crack growth of area $dA$ (energy release rate (ERR)) is larger than the critical fracture energy $G_c$. $G$, is evaluated using the $J$ integral (Rice (1968)) which reads:

$$G = J = \left[ Ud\gamma - T \cdot \frac{du}{dx} ds \right]$$

where $\Gamma$ is the path surrounding the crack tip, $T$ is the traction vector along the path $\Gamma$, $u$ is the displacement vector, $ds$ is an element of arc-length along $\Gamma$, and $U$ is the strain energy density function. Alternatively, the ERR can be directly evaluated through analytical or numerical derivative of the total potential energy:

$$G = - \frac{\partial \Pi}{\partial A} = - \frac{1}{b} \frac{\partial \Pi}{\partial a}$$

where $\Pi$ is the total potential energy that equals the internal strain energy plus the potential of the external loads and $a$ is the length of the interfacial crack and the debonded region. A comparison between the two methods and their applicability to the evaluation of ERR associated with the initiation and growth of the edge debonding using a linear theory appears in Rabinovich (2004b). In the numerical study presented next, the ERR associated with the growth of the inner debonding under geometrically nonlinear conditions is examined.

**NUMERICAL STUDY**

The geometry, mechanical properties, and loading scheme of the examined beam appear in Figure 2. The response of the strengthened beam with a 50 mm long debonded region at midspan throughout the loading process is examined first. Next, the structural response through the debonding growth process under a constant load level of $q'=q=35kN/m$ is examined.
The nonlinear behavior of the strengthened beam under the uplifting load appears in Figure 3. The development of the peak deflections in the RC beam and the FRP strip with the load and the development of the "relative deflection parameter" and the "waviness parameter" that are defined by:

\[
\Omega_x = \int_0^L \left( w_{frp} - w^c \right)^2 dx / \int_0^L \left( w_{frp} + w^c \right)^2 dx ; \\
\Omega_{2w} = \int_0^L \left( w_{frp} - w^c \right)^2 dx / \int_0^L \left( w_{frp} + w^c \right)^2 dx
\]  

respectively, appear in Figures 3a-b. These curves reveal that buckling of the debonded FRP strip occurs at a load level of about 62 kN/m. This phenomenon is characterized by an increased out-of-plane deflection and waviness of the FRP strip, which are observed in Figures 3a and 3b. The development of the inplane stress resultant in the FRP strip at midspan appears in Figure 3c. This curve shows that due to the buckling of the FRP strip, the development of the axial compressive force in the strip at midspan is stopped. As a result, the bending moments due to loading beyond the buckling point are solely carried by the RC section. Figures 3d-e show the development of the peak vertical normal stresses and shear stresses near the interfacial crack tip with the growth of the external load. These curves reveal that the stress concentration near the edge of the debonded region is significantly affected by the buckling of the FRP strip and the nonlinear response of the structure. The rapid increase in the magnitudes of the shear and, especially, the vertical normal stresses in this region imply that the debonded region may tend to burst. This effect is examined using the fracture mechanics criterion in Figure 3f. The response of the structure in terms of the energy release rate due to growth of the 50 mm long debonded region appears in Figure 3f and reveals that the buckling of the FRP strip significantly amplifies the tendency of the debonded region to grow. In quantitative terms, the energy release rate is increased by up to 3 orders of magnitudes in the post-buckling stage. Adopting a characteristic value of \( G_c = 65 \text{ J/m}^2 \) (Rabinovitch and Frostig (2003)) this observation means that once the buckling load is reached, a further increase in the load would lead to a spontaneous growth of the interfacial debonding crack.

Another aspect of the response of the debonded structure is the stability of the crack growth process and the possibility of crack arrest. To study this aspect, the response of the structure during the debonding crack growth under a constant load level is studied. The curves showing the development of the peak vertical deflections, the waviness parameters, the axial force in the FRP strip at midspan, the peak values of the shear and vertical normal stresses, and the ERR with the growth of the debonded region under \( q^c = 35 \text{ kN/m} \) appear in Figure 4. These figures show that the debonded regions shorter than about 65 mm do not buckle and the magnitudes of the localized deflections, the stress near the debonded region, and the ERR are not significant. However, beyond this critical crack length, buckling of the FRP strip is observed. This effect is associated with a relative vertical deflection between the FRP strip and the RC beam (Figure 4a) and a reduction in the compressive force carried by the FRP strip (Figure 4c). Resulting from the reduction in the ability of the FRP strip to carry compressive forces, the portion of the global bending moment carried by the RC beam is increased and the complementary portion carried in the form of a couple of axial forces in the FRP strip and the RC beam is decreased. In other words, the development of the debonded region reduces the composite action of the existing beam and the bonded strip. It also yields a considerable increase in the vertical normal and shear stresses in the adhesive layer near the debonded region (Figures 4d and 4e). Hence, it clarifies that when the critical combination of load and debonded length is reached, the buckling of the debonded FRP strip has a great influence on the response of the structure.

The response in terms of the development of the ERR with the crack growth (Figure 4f) reveals a different and unique phenomena. This curve shows that once the debonded FRP strip has buckled, the ERR is significantly increased and, adopting the value \( G_c = 65 \text{ J/m}^2 \), the critical fracture energy is exceeded very fast. At the initial steps of the interfacial crack growth process, the ERR is further increased implying that the propagation process is an unstable one. At a crack length of about 105 mm, the ERR attains its maximal value. Beyond this point, it decays and at a crack length of about 185 mm it goes down below the critical fracture energy. In physical terms, this observation means that the crack growth process becomes stable, the debonding crack may be arrested, and the debonding process can stop. In other words, the ERR curve quantitatively defines three "critical lengths" for the debonding crack. The first critical length \( (a', c = 65 \text{ mm}) \) defines the buckling point under the prescribed
external load, the second \((a^2_{cr}=105 \text{ mm})\) defines the point beyond which the crack growth shifts from an unstable process to a stable one, and the third \((a^3_{cr}=185 \text{ mm})\) designates the point beyond which the crack may be arrested. This information is essential in order to quantitatively assess the performance of the strengthened beam under its entire loading envelope and in order to assure its safety.

**Figure 3:** External load versus structural response, \(a=50 \text{ mm}\): (a) peak vertical deflections (b) relative deflection and waviness parameters (c) inplane force in the FRP strip at L/2 (d) peak vertical normal stresses \((z_a=0)\) − adhesive-concrete interface; \((z_a=ca)\) − adhesive-FRP interface; (e) peak shear stresses (f) energy release rate.

**Figure 4:** Debonded length versus structural response, \(q^{cr}=-35 \text{ kN/m}\): (a) peak vertical deflections (b) relative deflection and waviness parameters (c) inplane force in the FRP strip at L/2 (d) peak vertical normal stresses \((z_a=0)\) adhesive-concrete interface; \((z_a=ca)\) adhesive-FRP interface; (e) peak shear stresses (f) energy release rate

**SUMMARY AND CONCLUSIONS**

In this paper, an analytical approach for the quantitative assessment of the debonding process in RC beams strengthened with externally bonded composite materials has been presented. The paper focuses on the behavior of beams with pre-existing debonded regions along the beam and on cases in which various loading scenarios yield compressive stresses in the bonded strip. The investigation has combined two analytical approaches. First,
the overall response of the strengthened beam, the localized buckling of the debonded FRP strip and the stress fields near the debonded region have been analyzed through a geometrically nonlinear high order model. Next, the fracture mechanics concept of the energy release rate has been adopted for the assessment of the tendency of the interfacial debonding crack to burst and of the stability of the debonded region growth process.

The analytical model has been adopted for the study of a reinforced concrete beam strengthened with an externally bonded FRP strip and subjected to an uplifting load. The overall and localized nonlinear response of the beam, the tendency of the debonded region to grow, and the stability of the debonding process have been investigated. The results have quantitatively shown that the buckling of the FRP strip significantly increases the tendency on the debonded region to grow. Tracking down the interfacial crack growth process reveals that under a constant load level, the energy release rate is considerable increased. However, after it attains its maximal value, it tends to decrease and it may be reduced to values that are below the critical fracture energy. Thus, crack arrest and stabilization of the interfacial debonding process is possible. The ability to quantitatively predict and describe this type of behavior is essential for the design of the strengthened beam, for the assessment of its ability to resist its loading envelope, and for the safe and reliable usage of the strengthened member.

REFERENCES

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314