A CONFINEMENT MODEL FOR FRP-CONFINED NORMAL AND HIGH-STRENGTH CONCRETE IN SQUARE AND RECTANGULAR SECTIONS

J. C. Lim 1 and T. Ozbakkaloglu 2

1, 2 School of Civil, Environmental and Mining Engineering, University of Adelaide, Australia.

ABSTRACT

The confinement of concrete with fiber reinforced polymer (FRP) composites can significantly enhance its strength and deformability. However, the effectiveness of FRP confinement on square and rectangular concrete sections, in which the concrete is non-uniformly confined, is much lower than the effective confinement of circular sections. To investigate the shape factors influencing the compressive behavior of FRP-confined concrete in square and rectangular sections, a database of existing test results was assembled. The database was then studied, together with a companion database consisting of the test results of FRP-confined concrete in circular sections, in order to capture the change in the effectiveness of confinement due to the change in the sectional shape. The combined database catalogued the compression test results of 1547 specimens with unconfined concrete strengths ranging from 6.2 to 169.7 MPa. It provided a significantly extended parameter space, thereby allowing clearer observations of the important factors influencing the compressive behavior of FRP-confined concrete in various sections. Based on the test database, a unified model for FRP-confined concrete in circular, square and rectangular sections was developed and presented in this paper.

KEYWORDS

Fiber reinforced polymer (FRP); Confinement; Compression; High-strength concrete (HSC); Square and rectangular sections; Stress-strain behavior.

INTRODUCTION

Axial compressive behavior of FRP-confined concrete has received significant attention over the last two decades, and it is now well understood that the confinement of concrete with fiber reinforced polymer (FRP) composites can substantially enhance concrete strength and deformability. As demonstrated in a recent review by Ozbakkaloglu et al. (2013), it is clear that the behavior of FRP-confined concrete in circular sections has been extensively studied, resulting in the development of over 80 stress-strain models. On the other hand, the behavior of FRP-confined concrete in square and rectangular sections received less attention. Furthermore, it was reported that in Ozbakkaloglu (2013b) the performance of the existing models applicable for square and rectangular sections (Shehata et al. 2002; Lam and Teng 2003; Harajli et al. 2006; Yan and Pantelides 2007; Youssef et al. 2007; ACI-440 2008; Ilki et al. 2008; Wei and Wu 2012) are not yet satisfactory in their predictions on the ultimate conditions of confined high-strength concrete (HSC). To address these gaps, an extensive database consisting of 484 test results of FRP-confined concrete in square and rectangular sections was first assembled from published literature. In addition, a second database of FRP-confined concrete in circular sections that covers 1063 test results (Lim and Ozbakkaloglu 2013; Ozbakkaloglu and Lim 2013) was employed. A new model that is applicable to circular, square and rectangular sections of FRP-confined NSC and high-strength concrete (HSC) columns that was developed on the basis of these databases, is presented in the second half of this paper.

THE EXPERIMENTAL DATABASES

In this study, a carefully prepared test database of FRP-confined square and rectangular concrete sections tested under axial compression is used in the development of a new model. The suitability of these results for the database was then assessed using a set of carefully established selection criteria to ensure the reliability and consistency of the database. Only monotonically loaded circular specimens with unidirectional fibers orientated in the hoop direction and specimen height-to-width ratios ($H/b$) less than 4 were included in the assessment database. Specimens containing internal steel reinforcement, partial FRP confinement or specimens were not included. This resulted in a final database size of 484 datasets collected from 37 experimental studies published between 1994 and middle of 2013. The database consisted of specimens confined by five main types of FRP materials [carbon FRP (CFRP); high-modulus carbon FRP (HM CFRP); ultra high-modulus carbon (UHM

...
CFRP); S- or E-glass FRP (GFRP); and aramid FRP (AFRP)] and two confinement techniques (wraps and tubes). 421 specimens in the database were FRP-wrapped, whereas 63 specimens were confined by FRP tubes. 380 of the specimens were confined by CFRP, 55 by GFRP, 44 by AFRP, three by HM CFRP, and two by UHM CFRP. The specimens included in the test database consisted of square and rectangular cross-sections with sectional dimensions that varied from 70 to 600 mm, with the majority of the specimens having sectional dimensions of 150 x 150 mm. 331 specimens in the database were square and 153 specimens were rectangular in cross-section. The unconfined concrete strength ($f'_{cu}$) varied from 10.0 to 110.8 MPa.

The results of FRP-confined concrete in circular sections were collected to form another database. Only monotonically loaded circular specimens with unidirectional fibers orientated in the hoop direction and aspect ratios ($H/D$) less than 3 were included in the assessment database. The details of the NSC and HSC components of the circular column database can be found in Ozbakkaloglu and Lim (2013) and Lim and Ozbakkaloglu (2013), respectively. The database of FRP-confined concrete in circular sections contained 1063 datasets assembled from 105 experimental studies. 910 specimens in the database were FRP wrapped, whereas 153 specimens were confined by FRP tubes. 619 of the specimens were confined by CFRP, 246 by GFRP, 116 by AFRP, 64 by HM CFRP, and 18 by UHM CFRP. The diameters of the specimens ($D$) included in the test database varied between 47 and 600 mm. The peak unconfined concrete strength ($f'_{cu}$) varied from 6.2 to 169.7 MPa.

**NEW MODEL FOR FRP-CONFINED SQUARE AND RECTANGULAR COLUMNS**

As an extension of a recently developed model by the Lim and Ozbakkaloglu (2013) for FRP-confined circular columns, the new model contains simple closed-form expressions and was developed using the combined test databases of confined concrete in circular, square and rectangular FRP-confined concrete specimens. The model is applicable to both NSC and HSC columns of unconfined concrete strengths of up to 120 MPa. Two sets of expressions are proposed for ultimate conditions of FRP-confined concretes. The first set was developed for specimens exhibiting stress-strain curves with ascending second branches and the second set was developed for specimens with descending second branches. To differentiate and categorize the specimens with ascending and descending second branches, a concept of threshold confinement level is introduced, which is discussed in the following section.

**Threshold Confinement Level**

Confinement stiffness threshold ($K_{io}$) is the minimum stiffness of FRP confining shell required by the confined concrete to exhibit a stress-strain curve with an ascending second branch. It was previously shown that the confinement stiffness threshold ($K_{io}$) changes with the unconfined concrete strength ($f'_{co}$) (Ozbakkaloglu and Akin 2012; Lim and Ozbakkaloglu 2013; Vincent and Ozbakkaloglu 2013). Based on the experimental database of FRP-confined concrete in circular sections, the relationship of confinement stiffness threshold ($K_{io}$) in terms of unconfined concrete strength ($f'_{co}$) is expressed as:

$$K_{io} = 73.7e^{0.027f'_{co}}$$  \(1\)

![Figure 1. Variation of ultimate stress ratio ($f'_{cu}/f'_{co}$) with threshold stiffness ratio ($k_{s1}K/K_{io}$)](image)

In square and rectangular concrete sections, the equivalent confinement stiffness provided by the FRP shell is defined as $k_{s1}K_{i}$, where $k_{s1}$ is the strength efficiency factor, and $K_{i}$ is the confinement stiffness calculated from Eq. 1. Fig. 1 shows the variation of the ultimate stress ratio ($f'_{cu}/f'_{co}$) of the FRP-confined square and rectangular concrete specimens with the threshold confinement stiffness ratio ($k_{s1}K/K_{io}$). A value of $k_{s1}K/K_{io}$ greater than 1 represents a specimen having an equivalent stiffness above the minimum threshold, for which an ascending
second branch is expected, and vice versa. In the proposed model, this boundary condition is used to distinguish and categorize the stress-strain curves of FRP-confined concrete.

\[
K_1 = \frac{2E_{cf}}{d_c}
\]  
(2)

In Eq. 2, \(d_c\) is the equivalent sectional dimension of square and rectangular sections, and it is defined by Eq. 3, where \(b\) and \(h\) are the width and depth of the section \((h \geq b)\). For a circular section, \(d_c\) is equal to the diameter \((D)\).

\[
d_c = \sqrt{\left(\frac{h^2 + b^2}{2}\right)}
\]  
(3)

### General Form of the Proposed Expressions

Eqs. 4 and 5 are proposed for the predictions of the compressive strength \((f'_{ce})\) and ultimate axial strain \((\epsilon_{cu})\) of FRP-confined concretes that exhibit stress-strain curves with ascending second branches. Eqs. 6 and 7 are proposed for the predictions of the ultimate axial stress \((f'_{cu})\) and strain \((\epsilon_{cu})\) of FRP-confined concretes that exhibit stress-strain curves with descending second branches.

\[
\text{If } k_1 K_i/K_{lb} \geq 1: \text{ stress-strain curves with ascending second branches}
\]

\[
f'_{ce} = f'_{co} + k_1 (K_i - K_{lb}) \epsilon_{h,rup}
\]  
(4)

\[
\epsilon_{cu} = c_2 \epsilon_{co} + k_2 K_{s2,des} \left(\frac{K_i}{f'_{co}}\right)^{0.9} \epsilon_{h,rup}^{1.35}
\]  
(5)

\[
\text{If } k_1 K_i/K_{lb} < 1: \text{ stress-strain curves with descending second branches}
\]

\[
f'_{cu} = f'_{co} - k_{1,des} (K_{lb} - K_{s1,des} K_i) \epsilon_{h,rup}
\]  
(6)

\[
\epsilon_{cu} = c_2 \epsilon_{co} + k_{2,des} K_{s2,des} \left(\frac{K_i}{f'_{co}}\right)^{0.9} \epsilon_{h,rup}^{1.35}
\]  
(7)

In the expressions, \(f'_{co}\) is the unconfined concrete strength, \(K_{lb}\) is the threshold confinement stiffness, \(K_i\) is the confinement stiffness, \(k_1\) and \(k_2\) are the strength and strain enhancement coefficients, \(k_{1,des}\) and \(k_{2,des}\) are the strength and strain efficiency factors, and \(\epsilon_{h,rup}\) is the hoop rupture strain. In Eqs. 6 and 7, extra subscripts ‘des’ are assigned to \(k_{1,des}, k_{2,des}, K_{s1,des}\) and \(K_{s2,des}\) to make distinctions that these factors were established for specimens that exhibit stress-strain curves with descending second branches. In Eq. 7, \(c_2\) is the concrete strength factor and is calculated using Eq. 8 proposed by Lim and Ozbakkaloglu (2013). The peak axial strain of unconfined concrete \(\epsilon_{co}\) is determined using the expression proposed by Tasdemir et al. (1998) (Eq. 9).

\[
c_2 = 2 - \left(\frac{f'_{co}^2 - 20}{100}\right) \quad \text{and} \quad c_2 \geq 1
\]  
(8)

\[
\epsilon_{co} = \left(-0.067 f'_{co}^2 + 29.9 f'_{co} + 1053\right) \times 10^{-6}
\]  
(9)

### Strength and Strain Enhancement Coefficients

The strength and strain enhancement coefficients \((k_1\) and \(k_2\)) in Eqs. 4 and 5 were established from specimens that exhibit stress-strain curves with ascending second branches. These values were established from the database of circular specimens, specifically for each confinement method (i.e., FRP-wrapped or -tube encased concrete) and fiber type (e.g., carbon, glass or aramid), with average values recommended as \(k_1 = 4.1\) and \(k_2 = 0.27\). The strength decay coefficient \((k_{1,des})\) and the strain enhancement coefficient \((k_{2,des})\) in Eqs. 6 and 7 were established from specimens that exhibit stress-strain curves with descending second branches in the database of circular specimens. The recommended average values are \(k_{1,des} = 4.5\) and \(k_{2,des} = 0.27\). A detailed breakdown of the specific values of \(k_1\) and \(k_2\) calculated for each confinement method and fiber type can be found in Lim and Ozbakkaloglu (2013b).

### Strength and Strain Efficiency Factors

The effectiveness of FRP confinement in square and rectangular concrete sections, in which the concrete is non-uniformly confined, is known to be significantly lower than the effectiveness of confinement in circular sections (Restrepo and DeVino 1996; Campione and Miraglia 2003; Lam and Teng 2003; Ozbakkaloglu and Oehlerls 2008; Ozbakkaloglu 2013a; Ozbakkaloglu 2013b). A number of early studies in FRP-confined square and rectangular concrete sections (Restrepo and DeVino 1996; Campione and Miraglia 2003) attempted to capture the reduced confinement effectiveness resulted from the non-uniform confining pressure arising from the arching actions of confinement reinforcement. In these studies, a shape factor \((k_s)\) was directly related to the ratio between the concrete area effectively confined by the arching actions \((A_s)\) and the gross concrete area of the cross-section \((A_g)\). In more recent studies (Wu et al. 2007; Wei and Wu 2012), the shape factor \((k_s)\) was established on the basis of experimental test results, regardless of the effective confinement area ratio \((A_s/A_g)\).
Results of the model assessment reported in Ozbakkaloglu (2013b) indicate that that models that established the shape factors using experimental test results (Wu et al. 2007; Wei and Wu 2012) outperform their counterparts that used the theoretical effective confinement area ratios ($A_e/A_0$). On the basis of this finding, the latter approach was adopted in the present study. The shape factors were developed separately for specimens exhibiting stress-strain curves with ascending and descending second branches.

**Stress-strain curves with ascending second branches**

Fig. 2(a) shows the variation of the strength efficiency factor ($k_{s1}$) of square specimens with the corner radius ratio ($2r/d_e$). As expected, the value of $k_{s1}$ increases asymptotically with an increase in the $2r/d_e$ ratio, until it reaches 1 in the case of a circular section (i.e. $2r/d_e = 1$). Fig. 3(a) shows the variation of the strain efficiency factor ($k_{s2}$) of square specimens with the corner radius ratio ($2r/d_e$). As illustrated in the Fig. 3(a), $k_{s2}$ also increases with an increase in the $2r/d_e$ ratio. The test data having $k_{s2}$ values that exceed 1 in the range of $2r/d_e$ ratio between 0.4 and 1.0, indicate that square and rectangular specimens with well-rounded corners can exhibit higher ultimate axial strains than companion specimens with circular cross-sections.

Figure 2. (a) Variation of strength efficiency factor ($k_{s1}$) with corner radius ratio ($2r/d_e$), and (b) variation of normalized strength efficiency factor ($k_{s1}/(2r/d_e)^{0.67}$) with sectional aspect ratio ($h/b$)

Figure 3. (a) Variation of strain efficiency factor ($k_{s2}$) with corner radius ratio ($2r/d_e$), and (b) variation of normalized strain efficiency factor ($k_{s2}/(2r/d_e)^{1.0}$) with sectional aspect ratio ($h/b$)

After establishing the influence of the corner radius on strength efficiency factor based on the square specimens (i.e $k_{s1} = (2r/d_e)^{0.67}$), the influence of the sectional aspect ratio ($h/b$) was then established from the combined datasets of square and rectangular specimens. To capture discrete the influence of the sectional aspect ratio ($h/b$), the strength efficiency factor ($k_{s1}$) was normalized to eliminate the established influence of corner radius. Fig. 2(b) shows the relationship between the normalized strength efficiency factor ($k_{s1}/(2r/d_e)^{0.67}$) and the sectional aspect ratio ($h/b$). As evident from Fig. 2(b), $k_{s1}$ decreases with an increase in $h/b$ ratio. Based on the relationships illustrated in Figs.2(a) and 2(b), the following expression is proposed for the strength efficiency factor ($k_{s1}$):
Using a similar approach, the influence of the sectional aspect ratio \((h/b)\) on the strain efficiency factor \((k_{s2})\) was established. Fig. 3(b) shows the relationship of between the normalized strain efficiency factor \((k_{s2}\frac{(2-2r/d_s)^2r/d_s}{2r/d_s})\) and the sectional aspect ratio \((h/b)\). The trend of Fig. 3(b) indicates \(k_{s2}\) slightly increases with an increase in \(h/b\) ratio. Based on the relationships illustrated in Figs. 3(a) and 3(b), the strain efficiency factor is defined as:

\[
k_{s2} = \left(2 - \frac{2r}{d_e}\right)^2 \left(\frac{2r}{d_s}\right) \left(\frac{h}{b}\right) \frac{0.22}{0.67}
\]

**Stress-strain curves with descending second branches**

Following the approach outlined for specimens having stress-strain curves with ascending second branches, the strength and strain efficiency factors \((k_{s1,des} \text{ and } k_{s2,des})\) of specimens with descending second branches were derived from the test database (Eqs. 12 and 13). It should be noted that Eqs. 12 and Eq. 13 are not applicable to specimens having \(2r/d_e\) ratio lower than 0.15, as the failure modes of these specimens were found to be inconsistent, with both premature and progressive failures commonly observed.

\[
k_{s1,des} = k_s \frac{2r}{d_e} \geq 0.15
\]

\[
k_{s2,des} = \left(2 - \frac{2r}{d_e}\right) \left(\frac{h}{b}\right) \frac{0.22}{0.67} \text{ for } \frac{2r}{d_e} \geq 0.15
\]

**Hoop Rupture Strain**

To establish the relationship of the hoop rupture strain of the FRP shell \(\varepsilon_{h,rup}\) and the ultimate tensile strain of the fiber \(\varepsilon_f\), the hoop rupture strain \(\varepsilon_{h,rup}\) is calculated through Eq. 14 using hoop strain reduction factor \(k_{hf}\) given in Eq. 15. The \(k_{hf}\) expression was developed by Ozbakkaaloglu and Lim (2013) using a large experimental database of circular FRP-confined NSC and HSC specimens. The expression captures the observed reduction in the hoop strain reduction factor \(k_{hf}\) with an increase in compressive strength of concrete \(f'_{cu}\) and elastic modulus of confining fibers \((E_f)\), and it is applicable to concretes with \(f'_{cu}\) up to 120 MPa and confined by any FRP type.

\[
\varepsilon_{h,rup} = k_{hf}\varepsilon_f
\]

\[
k_{hf} = 0.9 - 2.3f'_{cu} \times 10^{-3} - 0.75E_f \times 10^{-6} \text{ where } 100,000 \text{MPa} \leq E_f \leq 640,000 \text{MPa}
\]

**COMPARISONS WITH EXPERIMENTAL RESULTS**

Fig. 4(a) and 4(b) show the comparisons of the predictions of the proposed model with the experimental results of specimens exhibiting stress-strain curves with ascending second branches. Figs. 4(a) and 4(b) illustrate that the prediction of the proposed model is in close agreement with the test results, with AAEs of 10.8% and 20.7% for the strength enhancement ratio \((f'_{cu}/f'_{co})\) and strain enhancement ratio \((\varepsilon_{cu}/\varepsilon_{co})\), respectively. Due to page limits, it was not possible to include a section on the comparison of model performances, which would demonstrate the significantly improved performance of the proposed model over the existing models.
CONCLUSIONS

This paper has presented the results of an investigation into the axial compressive behavior of FRP-confined NSC and HSC in circular and rectangular sections. A large experimental test database consisting of 484 test results of FRP-confined concrete in square and rectangular sections has been collected. The database was augmented with another database of FRP-confined concrete in circular sections to create a combined database of 1547 axial compression test results for FRP-confined concrete specimens with unconfined concrete strengths ranging from 6.2 to 169.7 MPa. The combined databases provided a significantly extended parameter space, thereby allowing clearer observations to be made on the important factors that influence the behavior of FRP-confined concrete. A new design-oriented model, which was developed on the basis of these databases, is presented in this paper. The model is applicable to FRP-confined concrete in circular, square and rectangular sections with unconfined concrete strengths up to 120 MPa, and it incorporates the important factors identified from the close examination of the results recorded in the database. The model comparisons have demonstrated that the proposed model provides improved predictions of the ultimate conditions of FRP-confined concrete compared to the existing models.

REFERENCES


