PROGRESSIVE DEBONDING IN RC BEAMS SHEAR-STRENGTHENED WITH
FRP SIDE STRIPS IN HOGGING MOMENT ZONE

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ABSTRACT

RC beams shear-strengthened with externally bonded FRP strips on their sides usually fail in shear due to debonding of the FRP strips. The bond behaviour between the FRP strips and the concrete substrate therefore plays a crucial role in the failure process of these beams. Following our previous analytical study on progressive debonding in RC beams shear-strengthened with FRP strips in sagging moment zone, this paper presents an analytical study on the progressive debonding in RC beams shear-strengthened with FRP strips in hogging moment zone. The complete debonding process is analysed and the contribution of the FRP to the shear capacity of the beam is quantified. The validity of the analytical solution is verified by comparing its predictions with numerical results from a finite element analysis.

KEYWORDS

FRP side strips, RC beams, shear-strengthening, debonding, closed-form solution, hogging moment zone

INTRODUCTION

The shear resistance of reinforced concrete (RC) beams can be enhanced by external bonding of fibre-reinforced polymer (FRP) reinforcement. The FRP reinforcement can be bonded around the entire section (complete wrapping), to the two sides as well as the soffit of the beam (U-jacketing), and to the two sides of the beam only (side bonding) (Teng et al. 2002). Extensive research has been conducted on the shear strengthening of RC beams with externally bonded FRP reinforcement in the last decade, but some aspects of the behaviour of such strengthened beams are still not well understood (Chen 2010). Examples of these aspects include shear interaction between the different components of shear contributions, the effect of beam size, and the effect of shear span-to-depth ratio.

For RC beams shear-strengthened with FRP side strips or U strips, FRP debonding is typically the governing failure mode. In such beams, the shear interaction mainly exists between internal steel stirrup and external FRP strips (Chen et al. 2013a). To quantitatively assess the effect of shear interaction, it is necessary to understand the development of shear resistance contributions from both the external FRP and the internal steel shear reinforcements. Chen et al. (2010) numerically investigated the development of shear contribution of steel stirrups as the critical diagonal crack develops. Chen et al. (2013a) proposed empirical functions based on the numerical predictions. Monti et al. (2004) were probably the first to present a closed-form solution for the shear contribution of shear-strengthening FRP as a function of the shear crack width based on a number of simplified assumptions for both FRP side and U strips. Chen et al. (2012) developed a more rigorous closed-formed solution based on more realistic assumptions, leading to a shear strength model capable of considering adverse FRP-steel interaction (Chen et al. 2013a).

Both Monti et al.’s (2004) and Chen et al.’s (2012) solutions cater for shear-strengthening FRP strips in the sagging moment zone of RC beams (referred to as case I hereafter). In practice, shear-strengthening in hogging moment zone is also common, such as in continuous beams and cantilever beams (referred to as case II hereafter). The boundary conditions at the ends of the FRP strips, and/or the variation of the FRP bond length with respect to the shear crack are different for these two cases. Therefore, the debonding process and thus the
shear contribution of FRP strips can be different in these two cases. Limited experimental studies have also demonstrated that there exists a significant difference in shear strengthening effect between the two cases (e.g. Sheng et al. 2005; Higgins et al. 2012). However, almost all existing shear strength models have been developed based on analysis and test of sagging moment zones. The direct application of these models in hogging moment zone is thus inappropriate and can be unsafe (Zhou et al. 2009), which necessitates the development of shear strength models specially-tailored for Case II.

This paper presents a closed-formed solution for the development of shear contribution of FRP strips in hogging moment zone with the opening width of the critical shear crack. As in Chen et al. (2012), this closed-form solution is based on an analytical solution for the full-range behaviour of FRP-to-concrete bonded joints assuming a linear bond-slip relationship for FRP-to-concrete bonded interface (Fig. 1), and a linear crack shape. The linear crack shape assumption is adopted because it usually leads to the most conservative predictions on FRP shear contribution (Chen et al. 2013b). The closed-form solution provides valuable insight into the debonding process of shear-strengthened FRP strips in Case II, and presents an important step towards developing shear strength models suitable for Case II. The solution is verified by comparing its predictions with finite element (FE) predictions.

DEVELOPMENT OF FRP SHEAR CONTRIBUTION WITH CRACK WIDTH

As with Case I (Chen et al. 2012), only the common shear tension failure mode is considered herein where the shear failure process of an RC beam shear-strengthened with FRP is assumed to be dominated by the development of a single critical shear crack at an angle $\theta$ from the beam longitudinal axis (Fig. 2). The key difference between Case II (hogging) and case I (sagging) is that in the former the flexural cracks start from the top (tension face) while in the latter they start from the bottom (also tensile) face. As with Case I, the lower end (i.e. the crack tip) of the shear crack at failure is assumed to be located at 0.1$d$ from the compression face (Fig. 2), with $d$ being the effective depth of the beam. The vertical distance from the bottom edge of the shear-strengthening FRP strips to the crack tip is assumed to be $h_b$, $\leq 0.1d$. Although the upper end of a shear crack is likely to be located at the tension face of the beam, only the portion of the shear crack between the crack tip and the centre of the steel tension reinforcement is considered in this study for reasons given in Chen and Teng (2003a, b). The intersection between the steel tension reinforcement and the critical crack is termed the ‘crack end’ (Fig. 2) herein and the shear crack between this crack end and the crack tip is termed the ‘effective shear crack’. The vertical distance from the top edge of FRP strips to the crack end is $h_t$, $\leq h_c$ which is the vertical distance from beam top to the centre of tension bars), and the height of the effective shear crack is 0.9$d$ which is equal to the effective height of FRP (i.e. $h_{f,e}$) when FRP is bonded to the full height of the beam sides, (i.e. $h_{f,e}=0.9d$).

The shear contribution of FRP can be evaluated as in Chen and Teng (2003a, b):

$$V_f = 2f_{r,s}w_f h_{f,e} \left( \frac{\cot \theta \cot \beta}{s_f} \right)$$

(1)

where $f_{r,s}$ is the effective (average) stress in the FRP intersected by the critical shear crack; $w_f$ is the width of individual FRP strips perpendicular to the fibre direction; $s_f$ is the centre-to-centre spacing of FRP strips measured along the longitudinal axis; $t_f$ is the thickness of FRP strips; $\beta$ is the angle between the fibre direction and the beam longitudinal axis. For continuous FRP sheets, $w_f = s_f \sin \beta$.

The development of the effective stress in FRP $f_{r,s}$ with shear crack width can be derived following Chen and Teng (2003a, b) as

$$f_{r,s} = \frac{\int \sigma(z)dz}{h_{f,e}} = \frac{D_{sf}f_{r,\text{max}}}{h_{f,e}}$$

(2)
where $\sigma(z)$ is the stress in the fibre direction of the FRP intersected by the critical shear crack at a coordinate $z$, $D_{frp}$ is the FRP stress distribution factor as defined by Chen and Teng (2003a, b), and $f_{max}$ is the maximum stress in all the FRP intersected by the effective shear crack which can be determined from the maximum bond strength of the FRP strips crossed by the effective shear crack.

The effect of shear crack shape on the shear contribution of FRP has been explored by Chen et al. (2013b). It was found that a linear crack width variation is the most critical among various crack width shapes studied. Such a linear shear crack shape is considered in this study. The crack width can be expressed as

$$w(z) = \frac{z - w_{max}}{h_{frp}}$$

(3)

where $w_{max}$ is the maximum shear crack width at the crack end; and $z$ is the vertical upward-coordinate starting from the crack tip (Fig. 2). Based on this assumption, it can be shown that FRP debonding always initiates at the crack end (Chen et al. 2010). For the present analytical solution, it is assumed for simplicity that complete debonding of individual FRP strips always occurs in a sequential manner from the crack end to the crack tip.

As the crack width increases, the bond area of the FRP intersected by the critical shear crack can generally be divided into three zones: a debonded zone where the FRP has debonded from the concrete; a mobilized zone where the FRP-to-concrete interface has entered the softening state, and an intact zone where both the interfacial shear stress $\tau$ and the interfacial slip $\delta$ are zero, as shown in Figure 2. These zones are divided by the debonding front and the softening front. The solution for the shear contribution of FRP side strips at different stages depends on the value of $h_{cos ec} \beta$ relative to the effective FRP bond length $a_{e}$ as follows.

When $h_{cos ec} \beta < a_{e}$

When $h_{cos ec} \beta < a_{e}$, the debonding process of FRP side strips can be divided into five key stages according to the value of $L_{m}$, including

(a) $0 \leq L_{m} < h_{cos ec} \beta$ ,

(b) $h_{cos ec} \beta \leq L_{m} < a_{e}$ ,

(c) $a_{e} \leq L_{m} < (h_{frp} + h_{frp})_{cos ec \beta} / 2$ and $h_{frp} < (h_{frp} + h_{frp}) / 2$ ,

(d) $a_{e} < L_{m} < (h_{frp} + h_{frp})_{cos ec \beta} / 2$ and $h_{frp} \geq (h_{frp} + h_{frp}) / 2$ ,

(e) $0 < L_{m} \leq a_{e}$ (Fig. 2). Here $L_{m}$ is the maximum mobilized bond length in the fibre direction (from the critical shear crack to the softening front where $\tau = \tau_{e}$ and $\delta = 0$) (Fig. 2), it increases from stage (a) to (c) and then decreases from stage (d) to (e) as debonding progresses. For these five stages, the expressions for the effective FRP stress $f_{frp}$ and the corresponding crack width at the crack end $w_{c}$ are given as follows. The detailed derivation can be found in Chen et al. (2013b).

(a) $0 \leq L_{m} < h_{cos ec} \beta$ (Fig. 2a)

During this stage, all the FRP strips are still bonded to the beam sides and $L_{m}$ is located at the crack end. Using the analytical solution for the full-range behaviour of FRP-to-concrete bonded joints presented in Chen et al. (2012), the effective stress in FRP $f_{frp}$ and the stress distribution factor $D_{frp}$ can be found as

$$f_{frp} = \frac{2G_{f}E_{c}}{\tau_{e}} D_{frp}$$

(4)

$$D_{frp} = \frac{L_{m} - \sin(L_{m}) \cos(L_{m})}{2 - 2 \cos(L_{m})}$$

(5)

The corresponding shear crack width at the crack end is

$$w_{c} = 2\delta_{f} \frac{1 - \cos(L_{m})}{\sin(\theta + \beta)}$$

(6)

(b) $h_{cos ec} \beta \leq L_{m} < a_{e}$ (Fig. 2b)

During this stage, some of the FRP strips near the crack end have debonded. Let the leftmost debonded fibre intersects the critical shear crack at a height $h_{db}$ below the tension steel reinforcement so $L_{m} = (h_{db} + h_{frp})_{cos ec \beta}$ (Fig. 2b), Eq. 4 is still valid and $D_{frp}$ and $w_{c}$ can be expressed as

$$D_{frp} = \frac{L_{m} - \sin(L_{m}) \cos(L_{m})}{2(1 - \cos(L_{m}))} \left[ 1 - \frac{h_{db}}{h_{frp}} \right]$$

(7)

$$w_{c} = 2\delta_{f} \frac{1 - \cos(L_{m})}{\sin(\theta + \beta)} \frac{h_{frp}}{h_{frp} - h_{db}}$$

(8)

(c) $a_{e} \leq L_{m} < (h_{frp} + h_{frp})_{cos ec \beta} / 2$ (Fig. 2c)

During this stage, the height of the debonded FRP area $h_{db}$ continues to increase with $L_{m} = (h_{db} + h_{frp})_{cos ec \beta}$. Eq. 4 is still valid but $D_{frp}$ and $w_{c}$ are given by
where $h_{gb}$ corresponds to the location where the slip of the FRP strip just reaches $\delta = \delta_f$ so that the mobilized bond length $L(h_{gb}) = a_u$ (Fig. 2c), which can be determined from:

$$h_{gb} = \frac{h_{f,c} - h_{gb}}{1 + \lambda(L_m - a_u)}$$  \hspace{1cm} (11)

(d) $a_u < L_m < (h_f, c + h_b + h_c) \cos \beta / 2$ (or $(h_{f,c}, c + h_b + h_c) < L_m < (h_f, c + h_b + h_c) - a_y \cdot \sin \beta)$. During this stage, the height of the debonded FRP area $h_{db}$ continues to increase while $L_m = (h_{f,c} + h_b - h_b) \cos \beta$ is decreased. The corresponding $D_{frp}$ and $w_c$ can be obtained from Eqs. (9) and (10) respectively.

(e) $L_m < a_u$ (or $h_{db} \geq h_{f,c} + h_b - a_y \cdot \sin \beta)$. During this stage, the height of the debonded FRP area $h_{db}$ continues to increase while $L_m = (h_{f,c} + h_b - h_b) \cos \beta$ is decreased. The corresponding $D_{frp}$ and $w_c$ can be obtained from Eqs. (7) and (8) respectively.

When $h_{b, \cos \beta} > a_y$, stages (e) and (f) above disappear (i.e. these two stages exist only if $h_{b, \cos \beta} < a_y$) and a stage (Fig. 2h) appears following stages (a), (b), (c) and (d). For this stage, the corresponding $D_{frp}$ and $w_c$ can be obtained from Eqs. (9) and (10) respectively.

**When $h_{b, \cos \beta} = a_y$.**

The debonding process of FRP side strips can also be categorized into five key stages, including (a) $0 \leq L_m < a_u$, (b) $a_u \leq L_m < h_{b, \cos \beta}$, (c) $h_{b, \cos \beta} \leq L_m < (h_f, c + h_b - h_c) \cos \beta / 2$, (d) $a_u < L_m < (h_f, c + h_b - h_c) \cos \beta / 2$ and (e) $0 < a_u < L_m$. The last three stages feature a debonded FRP height of $h_{db}$ measured from the crack end (see Figs. 5c-5g for reference).

Due to limit of space, detailed derivation and solutions for these stages are presented in Chen et al. (2013b).

It should be noted that the above solutions are applicable to both RC beams with rectangular section and RC beams with T-section. For example, the solutions for the T-section can be obtained by setting $h_{f,c} = 0.9d + h_c - h_f$ (where $h_f$ is the height of beam flange) and $h_c = 0$ when the beam web sides are fully covered with FRP.

**VERIFICATION OF THE ANALYTICAL SOLUTION.**

To verify the analytical solution, comparisons are made between its predictions and FE predictions herein. The FE model adopted by Chen et al. (2010) was employed. The FRP reinforcement was represented by 20 discrete FRP strips and modelled using the truss element [element T2D2 in ABAQUS (2010)]. The interface between FRP and concrete was modelled by the nonlinear spring element [element Spring 2 in ABAQUS (2010)] with $h_t = 250$mm. Both the linearly softening bond-slip model (Fig. 1) and the nonlinear bond-slip model (the simplified version) of Lu et al. (2005) were used in the FE analyses. The results are termed FEM1 and FEM2 respectively in Fig. 3.

Fig. 3 shows the relationship of shear force $V_f$ versus the crack end width $w_c$ for the case with $t_f = 0.11$mm. The analytical solution is almost identical to the FE prediction with a linearly softening bond-slip model (FEM1) for the ascending branch. After the shear force peaks, the FE model predicts stepwise drops, with each representing the complete debonding of one FRP strip. It is only in the final stage that the analytical solution deviates slightly from the FE predictions. This is because FRP strips are predicted by the FE model to debond in a much more abrupt way [see Chen et al. (2013b) for detailed discussions]. But this slight discrepancy should affect neither the applicability nor the accuracy of the analytical solution in any significant way.

The analytical $V_f$-$w_c$ curve for the same parameters but for the sagging moment zone, which has been obtained from Chen et al.’s (2012) solution, is also shown in Fig. 3 for comparison. The peak $V_f$ for the sagging moment...
zone is 20% higher than that for the hogging moment zone, mainly because of the larger bond length in the former than the latter in the early stages of debonding (see Chen et al. (2013b) for more details). The apparent difference of $V_f - w_c$ curve between sagging moment zone and hogging moment zone clearly demonstrates the significance of the present study.

Figure 2. Debonding process of FRP side strips for a thin concrete cover ($h_i \cos \beta \leq a_u$)
Fig. 3 also shows that the analytical solution overestimates by 3.3\% the maximum $V_f$ compared with the FE prediction using Lu et al.’s (2005) nonlinear bond-slip model (FEM2). This small overestimation can be tolerated for the significant benefit of adopting the simple linearly softening bond-slip model which enables the closed-form solution to be derived.

![Figure 3. Analytical solution versus FE predictions (h_f,e = 250 mm, t_f = 0.11 mm, h_f = 100 mm)](image)

**CONCLUSIONS**

A closed-form solution for the development of shear contribution of FRP side strips with the crack width in the hogging moment zone has been presented. The solution is based on an analytical solution for the full-range behaviour of FRP-to-concrete bonded joints and the assumption of linear shear crack shape. The solution has been verified by comparing its predictions with finite element predictions. An important benefit of the closed-form solution is that it can be used directly to derive the shear contribution of shear-strengthening FRP strips in the hogging moment zone of RC beams, and to evaluate the effect of shear interaction between external FRP strips and internal steel stirrups on the shear strength of RC beams shear-strengthened with FRP, as demonstrated in Chen et al. (2013b).

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