Shear Deformation of RC Beams Jacketed with Large Fracture Strain FRP in the Post-yielding Region

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ABSTRACT

The strengthening of the existing reinforced concrete (RC) members with a large fracture strain FRP has been successfully developed in order to enhance the shear strength and ductility of members under serious seismic forces. Although the models for predicting deformation have existed in many specifications, the design methods have not clarified the shear deformation precisely. In post shear cracking region, the experimental results indicate that the shear deformation mechanism significantly increases up to the failure of the member. Therefore, the deformation components are investigated in the plastic hinge region of RC beams with and without FRP jacketing. Based on the truss analogy, the method to predict the shear deformation is suggested at the post-yielding and at high ductility levels. In the model, the method of estimating the shear deformation takes into account the yielding of reinforcement and the reduction of compression strut angle due to the development of shear crack. To verify the application of the proposed model, measurements of Linear Variable Differential Transformers (LVDTs) and the Image analysis are evaluated for the tested beams in the critical plastic hinge region. Conclusions are drawn regarding the total deformation separating into the flexural and shear deformation components at each displacement level. Diagrams resulting from the proposed shear deformation model are also presented, indicating that the shear deformation mechanism becomes critical for beams with a short span to depth ratio.

KEYWORD

PET, shear deformation, Image analysis, truss model
1. INTRODUCTION

Worldwide attention has emphasized more on shear strengthening of deficient structures by Fiber-Reinforced Polymers (FRP). This is due to the various advantages of FRP such as high corrosion resistance and better long-term durability. Conventional strengthening materials such as steel always suffer from the corrosion problem which increases life cycle cost. However, the brittleness of FRP has been a major obstacle and reduces the ductility of the structure especially under seismic effects. To overcome the early breakage of the fiber and increase ductility, the use of a new type of fiber with high fracturing strain, PET (Polyethylene Terephthalate) has been developed [1,4].

Currently the deformation models regarding to the design for reinforced concrete (RC) structures are implemented in JSCE structural specification [2]. However, shear deformation model is not developed precisely due to the complications. According to previous studies [3,5], it is proved that shear deformation significantly would affect structural performance after the development of shear crack. One of effective models to estimate shear deformation is the truss mechanism with considering the shortening of concrete strut and elongation of steel tie. The current shear deformation model [5] is necessary to be extended to the post-yielding region in order to obtain precise prediction. Therefore, in this study, the method of estimating shear deformation is developed. The proposed model also accounts for the yielding of reinforcement and the reduction of compression strut angle due to the development of shear crack.

Considering the size effect, it is found that the development of cracking patterns depends highly on shear span to depth ratio. Therefore, in the experimental program, three RC beams with different shear span to depth ratio were tested and the image analysis was applied to measure shear deformation accurately. In this study, a shear deformation model is proposed to predict the precise shear deformation of RC members with and without FRP jacketing. The proposed model is taken into account the yielding of reinforcement. Another main issue is to investigate the influence of shear span to depth ratio on shear deformation. This paper presents the applicability of the proposed model by comparing with the experimental results of RC members with and without FRP jacketing.

2. EXPERIMENTAL PROGRAM

2.1 Materials

To investigate the influence of the shear span to depth ratio (a/d), the experimental program was conducted and material properties are described in this session.

(1) Concrete

The materials used in the concrete mixes were ordinary Portland cement, fine aggregates and coarse aggregates. The compressive concrete strength at 28 days was designed to be 27 MPa using maximum 20-mm coarse aggregate sizes.

(2) Steel reinforcement

Deformed steel reinforcement with 29-mm diameter was used for longitudinal reinforcement and its yield strength was 366 MPa. Stirrup was deformed steel reinforcement with 10-mm diameter and the yield strength was 406 MPa. The characteristics of reinforcement are summarized in Table 1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Elastic modulus GPa</th>
<th>Yield strength MPa</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>D29</td>
<td>159</td>
<td>366</td>
<td>Longitudinal</td>
</tr>
<tr>
<td>D10</td>
<td>170</td>
<td>406</td>
<td>Stirrup</td>
</tr>
</tbody>
</table>

(3) Fiber sheet

The fiber sheet used in this research was Polyethylene Terephthalate (PET). The manufacturing thickness of PET fiber sheet was 0.841 mm. The characteristic of PET fiber are listed in Table 2.

<table>
<thead>
<tr>
<th>Type</th>
<th>Thickness mm</th>
<th>Density g/m²</th>
<th>Elastic modulus GPa</th>
<th>Tensile strength MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>PET</td>
<td>0.841</td>
<td>1161</td>
<td>10</td>
<td>740</td>
</tr>
</tbody>
</table>

2.2 Instrumentation and details of specimens

In experimental program, three RC beams jacketed by PET fiber were designed with different shear span to depth ratio. The details of specimens identified as SC1-SC3 are summarized in Table 3. All specimens were performed under a monotonic three-point bending load and characteristics of tested specimens are illustrated in Fig. 1. The loading method was a load control in SC1 and SC2, while a displacement control was applied in SC3.
Table 3 Details of specimens

<table>
<thead>
<tr>
<th>No.</th>
<th>fiber type</th>
<th>$f'_c$ (MPa)</th>
<th>$a/d$</th>
<th>$p^{lt}_{l}$</th>
<th>$p^{st}_{w}$</th>
<th>$p^{st}_{f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC1</td>
<td>PET</td>
<td>34.5</td>
<td>5.6</td>
<td>1.53</td>
<td>0.48</td>
<td>1.12</td>
</tr>
<tr>
<td>SC2</td>
<td>PET</td>
<td>34.6</td>
<td>2.8</td>
<td>1.53</td>
<td>0.48</td>
<td>1.12</td>
</tr>
<tr>
<td>SC3</td>
<td>PET</td>
<td>32.3</td>
<td>1.4</td>
<td>1.53</td>
<td>0.48</td>
<td>1.12</td>
</tr>
</tbody>
</table>

*1 longitudinal reinforcement ratio, *2 stirrup ratio, *3 fiber sheet ratio

![Fig. 1 Characteristics of specimens](image)

Deformations at mid-span and support were measured by Linear Variable Differential Transformers (LVDTs) while shear deformation was measured by an image measurement method. Strain gauges were mounted on surface of steel and fiber in order to measure strain of longitudinal reinforcement, stirrups and fiber sheet in shear cracking location. An instrumentation and image measurement are shown in Fig. 2.

![Fig. 2 Instrumentation and image analysis](image)

2.3 Measurement of shear deformation

To measure shear deformation, reinforced concrete specimen was divided into a truss unit illustrated by target coordinates, A, B, C and D (Fig. 3a). The height and length of a truss unit ($h_1$ and $l$) was 85 and 100 mm, respectively. Total length of the truss system is equal to the depth of specimen ($1D = 300mm$) as shown in Fig. 3b. Using four digital cameras with high resolution, a movement of target coordinates to the arbitrary coordinates, $A'$, $B'$, $C'$ and $D'$, was captured every 30 seconds.

![Fig. 3 Truss mechanisms](image)

It is found that the influence of chord rotation should be taken into account in the estimation of shear deformation for a specimen with a long shear span length. Based on the method proposed by Massone and Wallace [3], the deformed configurations are as shown in Fig. 4. The un-deformed rectangular shape is represented by dotted lines while the pure shear deformation is represented by a shaded area. The total deformation corresponding to combined flexural and shear deformation is shown in solid lines. In case of shear deformation without flexural effect, the center of rotation locates at the mid-point of deformed truss section.

![Fig. 4 Deformed configurations](image)

Considering the movement of the coordinates, the diagonal lengths, $d_{1\text{meas}}$ and $d_{2\text{meas}}$, can be obtained by the law of cosines. The diagonal lengths corresponding to the pure shear deformation are represented as $d_{1\varepsilon}$ and $d_{2\varepsilon}$, respectively. Shear deformations at top and bottom are represented as $v_{1\varepsilon}$ and $v_{2\varepsilon}$. Therefore, the average shear deformation ($v_i$) can be obtained by subtracting the flexural deformation ($v_f$) from the total deformation ($v_{total}$) as follows:

$$v_i = v_{total} - v_f$$ (1)
where \( u_1, u_2 = \) the lateral deformation at the top and bottom of truss unit

The contribution of the flexural deformation \((v_f)\) attributes to the rotation of tension and compression chords, AD and BC, respectively. In this study, the lateral deformation due to flexure \((v_f)\) is assumed to be equal for each section. The flexural deformation is calculated as follows:

\[
v_f = a \theta l
\]

An \( a \)-value describes a distance from the top to the center of rotation and was assumed as 1 in this study. It should be noted that the center of rotation is at the bottom of the element in this assumption. Therefore, the rotated angle of chords can be calculated by:

\[
\theta = \frac{u_1 - u_2}{h_t}
\]

### 3. EXPERIMENTAL RESULTS

#### 3.1 Shear capacity and failure mode

The shear capacity and total deformation curves of SC1-SC3 are shown in Fig. 4. Specimen SC1 with shear span to depth ratio of 5.6 represents a slender beam while shear span to depth ratio specimen SC2 was 2.8 representing a normal beam. Moreover, specimen SC3 was designed as a deep beam with shear span to depth ratio of 1.4. All tested specimens failed in flexure due to the strength enhanced by PET fiber contribution.

To consider shear force components in RC member, shear forces carried by stirrup and fiber sheet \((V'_{s+f})\) are obtained from measured strain gauges along shear cracking. Therefore, shear force carried by concrete \((V_c)\) is calculated by subtracting total shear force by stirrup and fiber shear forces. The ultimate shear forces of SC1-SC3 are 100.3, 174.6 and 342.5 kN, respectively. From the experimental results, specimen SC3 provided the highest shear capacity due to a short shear span while SC1 showed the lowest shear capacity.

#### 3.2 Cracking patterns

After removing fiber sheets, the cracking patterns of specimen with different shear span to depth ratio were observed as shown in Fig. 5. The thicker lines represent the wider crack opening. In all specimens, the flexural-shear cracks occurred and propagated from bottom to top of specimen. It can be seen that flexural-shear cracks are accounted for the major crack of specimens SC1 and SC2 in a critical region. On the contrary, diagonal shear crack remarkably developed in specimen SC3 and the shear crack opening in specimen SC3 distributed more widely than that of specimen SC1 and SC2.
3.3 Deformation components

In general, deformation components of RC beam consist of flexural and shear deformation. Therefore, the total deformation is the combination of deformation due to flexure and shear. Based on the image analysis, the shear deformation was calculated by Eqs. (1)-(4). The flexural deformation can be estimated by subtracting the shear deformation from the total deformation. To examine the influence of shear span to depth ratio to deformation components, two types of deformation are shown in Fig. 6.

![Fig. 6 Deformation components](image)

**Note:**
- I - No cracking
- II - Flexural cracking
- III - Shear cracking
- \( \delta_f \) = shear deformation
- \( \delta_s \) = flexural deformation
- \( \delta_{fs} \) = additional flexural deformation

Based on the cracking patterns, deformation can be separated into three stages which are stages I) before cracking, II) after flexural cracking and III) after shear cracking. At the stage before cracking, the flexural deformation (\( \delta_f \)) corresponds to the elastic theory and contributes approximately 55-70% of the total deformation. For shear deformation (\( \delta_s \)), its contribution was smaller than that of flexural deformation in pre-cracking region. After occurrence of flexural cracking, flexural stiffness reduced due to decrease of effective concrete area. Consequently, the flexural deformation increased further afterward. However, it is observed that the shear deformation slightly increased in this stage.

Following the onset of shear cracking, flexural deformation of specimen SC1 continuously increased whereas shear deformation gradually developed. This is because the number of shear crack was less than that of flexural crack. Therefore, the contribution of shear deformation of specimen SC1 at ultimate is 18% of total deformation. In specimen SC2, flexural and shear deformation increased further in post shear cracking stage. At ultimate, shear deformation of specimen SC2 contributed approximately 36% of total deformation. In specimen SC3, shear crack remarkably developed from bottom of support to top of the specimen. It is also evident that shear cracking propagated rapidly with greater applied load. As a result, the shear deformation of SC3 accounted for approximately 62% of the total deformation.

3.3 Tension shifting effect

It is well understood that shear cracking induces the stress on the tension reinforcement, which accounts for greater deformation. This phenomenon is known as tension shift, which can be defined as the influence of shear cracking on flexural deformation. The increase in the tension stress due to tension shift can be expressed in terms of an additional tension force in the tension reinforcement (\( \Delta T \)). Therefore, the tension force increases when the shear force (\( V \)) is larger than the diagonal shear crack force (\( V_{crack} \)) as follows:

\[
V \leq V_{crack} ; \quad \Delta T = 0
\]

\[
V > V_{crack} ; \quad \Delta T = V_{nf} \left( \frac{c \alpha t - c \alpha x}{2} \right)
\]

It should be noted that the additional tension force induces greater tensile stress, even though the sectional moment (\( M \)) remains the same after shear cracking. This is because the diagonal compression strut force and the tension force in the shear reinforcement cancel the additional moment induced by the additional tension force in the tension reinforcement.

According to the tension shift effect, the additional tension force influences the increment of strain in the tension reinforcement. Consequently, the additional curvature (\( \Delta \phi \)) is produced by the incremental strain, which leads to a larger flexural deformation. In the shear cracking region or the plastic hinge region (\( l_p \)), accounting for the tension shift effect, the additional flexural deformation (\( \delta_{fs} \)) is obtained from the double integration along the length of member as follows:

\[
\delta_{fs} = \int \int \Delta \phi dz dx
\]

Knowing the neutral axis depth of a section (\( x \)) and additional strain of longitudinal reinforcement (\( \Delta \epsilon \)), the additional curvature can
be obtained by:

$$\Delta \phi = \frac{\Delta \epsilon}{x} \tag{8}$$

It can be seen from Fig. 6 that the additional flexural deformation ($\delta_{fl}$) of specimens SC1 and SC2 accounted for 5-10% of total deformation after shear cracking. For specimen SC3, the additional flexural deformation induced by shear crack substantially increased after shear cracking and contributed approximately 23% of total deformation at ultimate.

4. SHEAR DEFORMATION MODEL

4.1 Shear deformation before shear cracking

The shear deformation of RC members before shear cracks should be calculated based on beam elastic theory.

$$\delta_s = \frac{6}{5} \int \frac{V}{G_c A_c} \, dx \tag{9}$$

The shear stiffness of concrete is represented as $G_c = (E_c / [2(1+\nu_c)])$ and Poisson's ratio of concrete is $\nu_c$. An effective area of concrete ($A_e$) is separated into before and after flexural cracking as follows [5]:

before flexural cracking:

$$A_e = A_g$$

after flexural cracking:

$$A_e = A_g \left( \frac{M_{cr}}{M_{max}} \right)^3 + A_{ct} \left[ 1 - \left( \frac{M_{cr}}{M_{max}} \right)^3 \right] \tag{10}$$

where $A_g$ and $A_{ct}$ are a gross and cracked cross-sectional area, $M_{cr}$ and $M_{max}$ are cracking and maximum moment at loading point under total shear force ($V_{total}$).

4.2 Shear deformation after shear cracking

After shear cracking, the shear deformation is considered separately for the case before and after yielding of tension and shear reinforcement. The truss mechanism model, originally proposed by Ueda et al. [5], for predicting the shear deformation before the yielding of reinforcement is extended to the post-yielding range.

Before yielding of reinforcement

In Fig. 7, a truss analogy depicts with the diagonal compression stress inclined at an angle ($\theta$) and the transverse reinforcement aligned at an angle ($\alpha$). The shortening of the compression strut and elongation of the tension tie are $\Delta I_{s,c}$ and $\Delta I_{s,t}$.

![Fig. 7 Shear deformation model](image)

The shear deformation is induced by both the shortening of the compression strut ($\delta_{st}$) and elongation of the tension tie ($\delta_{st}$) which is expressed in the following equations [5]. Moreover, it is evident that fiber contribution affect to the shear deformation since the secant stiffness of fiber with thickness ($E_\ell t_\ell$) provide larger and stiffer cross-sectional area.

$$\delta_{st} = \frac{\Delta \epsilon_{s,c}}{\sin \theta} + \frac{V_{s,c}}{E_c (\cot \theta + \cot \alpha) \sin^4 \theta} \tag{12}$$

$$\delta_{st} = \frac{\Delta \epsilon_{s,t}}{\sin \alpha} - \frac{V_{s,t}}{E_\ell \left( \cot \theta + \cot \alpha \right) \sin^4 \theta} \tag{13}$$

where

- $b$ = width of cross section
- $t_\ell$ = thickness of fiber sheet
- $E_c$ = secant modulus of concrete (It should be noted that the secant modulus corresponds to stress-strain value at any specified loading level)
- $E_\ell$ = secant modulus of web reinforcement
- $A_e$ = cross-sectional area of web reinforcement
- $A_{c,o}$ = cross-sectional area of surrounding effective concrete in tension after shear crack $= \frac{A_{c,o} f_{ct}}{f_t}$
- $A_{c,o}$ = cross-sectional area of surrounding effective concrete in tension $= A_{c,o} \left( \frac{V_{crack}}{V_{total}} \right)^3$
- $f_{ct}$ = yield strength of shear reinforcement
- $f_t$ = tensile strength of concrete
- $V_{crack}$ = shear force after diagonal shear cracking

suggested in JSCE-2002 standard specification.
\[ V_{\text{total}} = \text{total shear force} \]
\[ V'_{rsf} = \text{shear force carried by stirrup and fiber sheet} \]

In addition, the strut angle can be obtained by using nominal shear stress, \( \nu = V_{\text{total}}/bd \). In the model, the strut angle represents the single shear crack angle based on main shear crack as shown in Fig.5. According to Ueda et al.'s formula [5], the equations are shown as follows:

\[ \theta = -\alpha_c (\nu - \nu_c)^2 + \theta_c, \text{ for } \nu_c \leq \nu < 1.7\nu_c \]  
(14)
\[ \theta = \theta_c \left( \frac{1.7\nu_c}{\nu} \right)^a, \text{ for } 1.7\nu_c \leq \nu \]  
(15)

where
\[ \theta = 3.2 \left( \frac{a}{d} \right)^4 + 40.2, \text{ for } a/d \leq 1.5 \]  
(16)
\[ \theta_c = -\alpha_c (1.7\nu_c - \nu_c)^2 + \theta_c_{0} \]  
(17)
\[ \nu_c = 0.9\nu_c \]  
(18)
\[ \nu_c = 0.2 (f'_c)^3 \left( 100\rho_s \right)^{0.3} (1000/d)^{0.04} \left( 0.75 + \frac{1.4}{a/d} \right) \]  
(19)
\[ \alpha_c = 0.4 \left( \frac{a}{d} \right)^3 + 2.9 \]  
(20)
\[ \beta = \left( 0.7 - 32\sqrt{\rho_c} \right) \frac{a}{d} \]  
(21)

After yielding of reinforcement

The current method to calculate the shear deformation mentioned previously does not consider the influence of yielding of reinforcement. In this study, therefore, the model is extended to the post-yielding region. It is also evident that the crack inclination angle drops after the yielding of reinforcement. This is because a crack growth is longer in order to compensate for the reduction of resistant force due to yielding. In the truss mechanism, this crack angle can be referred as a strut angle. To improve the applicability of the model, a modified strut angle taking into account of yielding of reinforcement is obtained for the better correlation in shear deformation between the experiment and the model. According to the experimental results, the strut angle decreases after the yielding of reinforcement. Moreover, the reduction of remaining concrete strength (\( f'_c \)) also causes the decrease of strut angle. For the proposed strut angle equation, the secant stiffness modulus of reinforcement (\( E_v, E_w \) and \( E_f \)) and remaining concrete strength (\( f'_c \)) are included. By applying a nonlinear regression approach, the proposed strut angle equation is expressed as follows:

\[ \theta = \left( -0.3 \ln A^2 + 4.4 \ln A - 10.74 \right) \left( 0.4 \ln B^2 - 4 \ln B + 12.9 \right) \]  
\[ \left( -0.8 \ln C^2 + 4 \ln C - 1.5 \right) \left( 1 + (a/d)^2 \right) \]  
(22)

where
\[ A = \rho_c E_v, B = \rho_w E_w + \rho_f E_f \] and \( C = f'_c \)

5. MODEL VERIFICATION

5.1 General description of referred testing

In this study, the RC beams tested by the authors and Senda [4] are taken to verify the shear deformation model. The referred specimens were performed under monotonic loading and failed in either shear or flexure. The yielding of reinforcement was observed in all the specimens. The details of specimens tested by Senda are summarized in Table 4.

<table>
<thead>
<tr>
<th>No.</th>
<th>fiber</th>
<th>type</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>a/d</th>
<th>( p_s ) %</th>
<th>( p_{w+} ) %</th>
<th>( p_f ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP1</td>
<td></td>
<td></td>
<td>250</td>
<td>270</td>
<td>2.50</td>
<td>4.22</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>SP2</td>
<td>PET</td>
<td></td>
<td>250</td>
<td>270</td>
<td>2.50</td>
<td>4.22</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>SP3</td>
<td>PET</td>
<td></td>
<td>250</td>
<td>270</td>
<td>2.50</td>
<td>4.22</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>SP4</td>
<td>PET</td>
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<td>250</td>
<td>270</td>
<td>2.50</td>
<td>4.22</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td>SP5</td>
<td>PET</td>
<td></td>
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<td>270</td>
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<td>4.22</td>
<td>0.17</td>
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<td>SP6</td>
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<td>0.17</td>
<td>0.45</td>
</tr>
</tbody>
</table>

\(^1\) Longitudinal reinforcement ratio, \(^2\) stirrup ratio, \(^3\) fiber sheet ratio

5.2 Shear deformation

The calculated shear deformation at mid-span are compared with that of testing program. Fig. 8 shows shear force and shear deformation curves of some specimens. It can be seen that the shear deformation corresponding to the proposed model correlates well with that of tested result.

As explained previously, the experimental and calculated results in terms of shear deformation can be separated into three stages based on cracking patterns. For stages before cracking and after flexural cracking, calculated shear deformation agree well with that of experiment. However, the shear deformation model overestimates shear deformation of specimen SC3 with low shear span to depth ratio. At the stage where specimen nearly fails, calculated shear deformation overestimates in SC3. One of the reasons is that shear cracking results in significant stiffness reduction in the model. Another reason is a large reduction of strut angle near the last loading state in SC3 in the model. Overall shear deformation obtained from the proposed model showed a good agreement with that of testing results.
The correlation between experiment and model is represented in terms of R-square value which is equal to 0.9874. Therefore, it can be concluded that shear deformation is able to predict shear deformation of reinforced concrete members with and without FRP retrofitting precisely.

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