Improvement of Debonding Bending Moment of Pre-tensioned CFRP Plates Bonded onto Steel Members

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ABSTRACT

Recently, some research reports on the applications of pre-tensioned CFRP plates on steel members have been published. As well as applying the bending moment, however, releasing the pre-tension of the CFRP plate provide high shear and peel stresses at the adhesive ends. Therefore, in strengthening of steel members with pre-tensioned CFRP plates, the CFRP plates tend to have easily debonding than non-pre-stressed one. In this study, to reduce its shear and peel stresses at the adhesive ends due to releasing the pre-tension of the CFRP plates, installing non pre-tensioned regions in the CFRP plate is proposed. To verify whether the proposed method is effective, theoretical investigation was carried out under the simple loading condition of pure bending. It is found that the shear stress at the adhesive ends by releasing pre-tension of CFRP plate becomes almost 0 in proposed method. Therefore, the applicable limit of bending moment and pre-tension for prevention of debonding of CFRP plate is improved compared with conventional pre-tension method. Additionally, a design method of required length of the non pre-stressed regions is also presented.

KEYWORD

CFRP plate, non pre-tensioned region, debonding bending moment, steel member
1. INTRODUCTION

Due to increasing of design load by specifications for highway bridges in Japan, some steel girders in bridges need to be strengthened. In order to reduce the total cost of construction, Carbon Fiber Reinforced Polymer, CFRP, plates which are light weight and can be easily attached by bonding, is used for upgrading of steel girders [1]. Moreover, retrofitting with pre-tensioned CFRP plate is employed as efficient enhancement so that the stress in steel member induced by dead load can be reduced by provided pre-stress. However, by providing the pre-tension into CFRP plate, debonding bending moment, which is the bending moment when the CFRP plate peels off, will be small [2]. In order to improve the decrease of debonding bending moment, assigning the non pre-tensioned regions at the adhesive ends is proposed in this research.

NOTATION

- $b$: width of CFRP plate
- $h$: thickness of adhesive layer
- $A_s$: cross-sectional area of steel member
- $A_v$: cross-sectional area of CFRP plate
- $A_{sv}$: cross-sectional area of steel-CFRP composite member
- $I_s$: moment of inertia of steel member
- $I_v$: moment of inertia of CFRP plate
- $I_{sv}$: moment of inertia of steel-CFRP composite member
- $E_s$: Young’s module of steel
- $E_v$: Young’s module of CFRP plate
- $E_a$: Young’s module of adhesive
- $G_v$: shearing module of adhesive
- $x$: distance from the center of steel member reinforced by CFRP plate
- $d_c$: distance between centroid of steel member and its bottom
- $d_v$: distance between centroid of CFRP plate and its upper surface
- $l_p$: half of pre-tensioned length
- $l_n$: non pre-tensioned length
- $y_s$: distance from the centroid of steel member
- $y_v$: distance from the centroid of CFRP plate
- $a_s$: distance between centroid of steel-CFRP composite member and that of steel member
- $a_v$: distance between centroid of steel-CFRP composite member and that of CFRP plate
- $N_s$: axial force in steel member
- $N_v$: axial force in CFRP plate
- $Q$: applied shear force
- $V_s$: shear force in steel member
- $V_v$: shear force in CFRP plate
- $M$: applied bending moment
- $M_s$: bending moment in steel member
- $M_v$: bending moment in CFRP plate
- $\tau_e$: shear stress in adhesive layer
- $\tau_{ep}$: shear stress in adhesive layer at pre-tensioned region ends
- $\tau_{en}$: shear stress in adhesive layer at adhesive ends
- $\sigma_{ye}$: peel stress in adhesive layer
- $\sigma_{yep}$: peel stress in adhesive layer at pre-tensioned region ends
- $\sigma_{yen}$: peel stress in adhesive layer at adhesive ends
- $\sigma_p$: principle stress in adhesive layer
- $\sigma_p$: debonding strength of adhesive
Generally, high shear and peel stresses in adhesive layer occur at the adhesive ends subjected to bending moment. Similarly, shear and peel stresses by releasing pre-tension of a CFRP plate are concentrated at the adhesive ends. In this paper, it is theoretically explained that the locations of stress concentrations in adhesive by releasing the pre-tension and by the bending moment can be separated each other by applying proposed method.

2. SOLUTIONS OF DIFFERENTIAL EQUATIONS FOR PRE-TENSIONED CFRP BONDED STEEL MEMBER

2.1 Shear Stress in Adhesive

Figure 1 shows a side view and cross section of a CFRP plate bonded steel member subjected to pure bending moment. Figure 2 illustrates an infinitesimal element of CFRP bonded steel member. As shown in this figure, the shear and the longitudinal forces in the adhesive layer are ignored in the theoretical analysis because the stiffness of adhesive is quite smaller than that of steel member or CFRP plate. Therefore, in this study, it is assumed that the adhesive just transfers shear and peel stresses, as shown in Fig.2. From the longitudinal equilibrium of the element in Fig. 2, the differential equations regarding the axial force in steel member, \( N_s \), are produced as:

\[
\frac{d^2N_s}{dx^2} - c^2 N_s = c^2 \frac{K}{a} \left( M + \frac{Z_1}{a} E_s I_s \varepsilon_{pre} \right) \quad (0 \leq x \leq l_p) \tag{1}
\]

\[
\frac{d^2N_s}{dx^2} - c^2 N_s = c^2 \frac{K}{a} M \quad (l_p \leq x \leq l) \tag{2}
\]

where,
\[
c = \sqrt{a^2bG/(hKZ_sI_e)}, \quad a = d_s + d_n, \quad Z_s = 1 + nA_e/A_s, \quad \tau_e = \sqrt{I_e/A_s}, \quad K = 1/(1 + Z_s r_e^2/a^2), \quad n = E_s/E_e, \quad l = l_p + l_n
\]

Providing the boundary conditions, which are \( dN_s/dx = 0 \) at \( x = 0 \), continuous conditions of \( N_s \) and \( \tau_e \) at \( x = l_p \) and \( N_s = 0 \) at \( x = l \), the axial force in steel member is given as:

\[
N_s = \begin{cases} 
A_1 \cosh(cx) \\
- \frac{K}{a} \left( M + \frac{Z_1}{a} E_s I_s \varepsilon_{pre} \right) & (0 \leq x \leq l_p) \\
A_2 \cosh\left(x(l_p-l)\right) & (l_p \leq x \leq l) \\
A_3 \sinh\left(x(l_p-l)\right) - \frac{K}{a} M & (l_p \leq x \leq l)
\end{cases}
\tag{3}
\]

### Table 1 Material properties of steel member, CFRP plate and adhesive

<table>
<thead>
<tr>
<th></th>
<th>Width [mm]</th>
<th>Thickness [mm]</th>
<th>( E_s ) [GPa]</th>
</tr>
</thead>
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<tr>
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<td>200</td>
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<tr>
<td>Web</td>
<td>184</td>
<td>5.5</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Width ( b ) [mm]</th>
<th>Thickness ( t_s ) [mm]</th>
<th>Half pre-tensioned length ( l_p ) [mm]</th>
<th>Pre-strain ( \varepsilon_{pre} ) [( \mu )]</th>
<th>( E_s ) [GPa]</th>
<th>( G_e ) [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Steel girder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
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<tr>
<td>(b) CFRP plate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td>(c) Adhesive</td>
<td>Thickness ( h ) [mm]</td>
<td>( E_s ) [GPa]</td>
<td>( G_e ) [GPa]</td>
<td>0.5</td>
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### Fig. 1 Steel member reinforced by pre-tensioned CFRP plate with non pre-tensioned regions

### Fig. 2 An infinitesimal element of CFRP plate bonded steel member

where,

\[
A_1 = \frac{1}{\cosh(cl)} \left( \frac{K}{a} \left( M + \frac{Z_1}{a} E_s I_s \varepsilon_{pre} \cosh(cl) \right) \right)
\]

\[
A_2 = \frac{\cosh(cl_p)}{\cosh(cl)} \left( \frac{K}{a} \left( M - \frac{Z_1}{a} E_s I_s \varepsilon_{pre} \sinh(cl_p) \tan(c_l_p) \right) \right)
\]
\[ A_3 = \frac{\sinh(cl_3)}{\cosh(cl_3)} K \left\{ M + \frac{Z}{a} E \, I, \varepsilon_{pl} \cosh(cl_3) \right\}. \]

Shear stress in adhesive layer, \( \tau_y \), is also given by substituting Eq.(3) into following equation.

\[ \tau_y = -\frac{1}{b} \frac{dN_y}{dx} \]  

(4)

2.2 Peel Stress in Adhesive

From the vertical equilibrium and moment balance around the centroid of the CFRP plate in Fig. 2, the differential equation regarding shear force in steel member, \( V_y \), is provided as:

\[ \frac{d^2 V_y}{dx^2} + 4 \omega^4 V_y = \frac{4 \omega^4}{Z_i} (Q + J \frac{dN_y}{dx}) \]  

(5)

where, \( \omega = \sqrt{bE_iZ_i/(4hE_i)} \), \( J = d - (Z_i - 1)d_j \).

The solutions of Eq.(5) are given by the boundary conditions, which are \( Q = 0 \) (pure bending condition), \( dV_y/dx = 0 \) at \( x = 0 \), \( V_y = 0 \) at \( x = 0 \) and \( x = l \), \( M_y = M \) at \( x = l \), continuous conditions of \( V_y \), \( M_y \), \( \sigma_{yx} \) and \( d\sigma_{yx}/dx \) at \( x = l \cdot p \):

\[ V_y = \begin{cases} B_1 \cosh(ax) \sin(ax) \\ + B_2 \sinh(ax) \cos(ax) \\ + C & (0 \leq x \leq l_p) \end{cases} \]

\[ \cosh(ax) \sin(ax) \\ + B_2 \sinh(ax) \cos(ax) \\ + C & (l_p \leq x \leq l) \]  

(6)

where,

\[ C = \frac{4}{4 + (c/\alpha)^2} \frac{cJK}{aZ_i} \frac{\sinh(cx)}{\cosh(cl)} \times \left\{ M + \frac{Z}{a} E \, I, \varepsilon_{pl} \cosh(cl) \right\} \]  

\[ (0 \leq x \leq l_p) \]

\[ \frac{4}{4 + (c/\alpha)^2} \frac{cJK}{aZ_i} \frac{1}{\cosh(cl)} \left[ M \sinh(cx) + \frac{Z}{a} \right] \]

\[ \times E, I, \varepsilon_{pl} \cosh(cl) \cosh(cl - x) \]  

\[ (l_p \leq x \leq l) \]

\[ B = D^{-1} F, \quad B = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \end{bmatrix}^T, \quad D \]

is the matrix corresponding to boundary condition, \( F \) is the vector corresponding to the applied moment and provided pre-tension, as described in appendix (A1) and (A2). The peel stress in adhesive layer, \( \sigma_{yx} \), is given by substituting Eq.(6) into the following equation.

\[ \sigma_{yx} = -\frac{1}{b} \frac{dV_y}{dx} \]  

(7)

3. DISCUSSION

3.1 Stresses in Steel Member and CFRP Plate

The stresses in the steel member and the CFRP plate are expressed as:

\[ \sigma_x = \frac{N_x}{A_i} + \frac{M_x}{I_i} y \]  

(8)

\[ \sigma_y = \frac{N_y}{A_i} + \frac{M_y}{I_i} y \]  

(9)

The bending moment in the steel member, \( M_y \), is derived as:

\[ M_y = \frac{1}{Z_i} (M + N_i a) - \frac{1}{4 \omega^4} \frac{d^4 V_y}{dx^4} \]

(10)

Figure 3 shows the stress distributions in the steel
member and the CFRP plate induced by releasing the pre-tension of the CFRP plate. As shown in Fig. 3, the stresses occur even in the non pre-tensioned region as well as the pre-tensioned region. The stresses in the steel member and the CFRP plate at $x=0$ converge to the plots of Eqs.(11) and (12), which is given by the design concept of the composite member of steel members with CFRP plate.

$$
\sigma_x = \frac{M}{I_x} y_x - \left( \frac{M_{pre}}{I_x} y_x + \frac{P_{pre}}{A_i} \right) \quad (11)
$$

$$
\sigma_y = \frac{M}{nL_y} y_y - \frac{1}{n} \left( \frac{M_{pre}}{I_y} y_y + \frac{P_{pre}}{A_i} - E_y \varepsilon_{pre} \right) \quad (12)
$$

where, $M_{pre} = P_{pre} a_i$, $P_{pre} = E_a \varepsilon_{pre} / n$, $y_x = y_x - a_i = y_x + a_i$.

3.2 Shear Stress and Peel Stress in Adhesive

The distributions of shear, peel and principal stresses in adhesive layer induced by releasing the pre-tension of CFRP plate are shown in Fig. 4. In this figure, dimensions and material properties of steel member, CFRP plate and adhesive listed in Table 1 are used. $\sigma_x$ is calculated by [3], [4]:

$$
\sigma_x = \frac{\sigma_{ye}}{2} \left( \frac{\sigma_{ye}}{2} \right) + \tau_{ep} \quad (13)
$$

For the CFRP plate with non pre-tensioned region, as shown in Fig. 4, the peel stress is so small that it has little effect on the principal stress. Additionally, the maximum shear stress can be reduced by installing the non pre-tensioned region. The peak of shear stress appears at $x=l_p$.

Significantly, the shear stress is almost 0 at the adhesive ends, where the peak of shear stress in the conventional CFRP plate appears.

The shear stress at $x=l_p$ or $x=l$ and the peel stress at $x=l_p$ or $x=l$, $\tau_{ep}$, $\tau_{en}$, $\sigma_{ye}$ and $\sigma_{yen}$, vary with non pre-tensioned length $l_n$, as drawn in Fig. 5. Assigning a enough length of $l_n$, $\tau_{ep}$ is only existing while the $\sigma_{yen}$, $\tau_{en}$ and $\sigma_{yen}$ converge to 0. Then, if $l_p$ becomes large, $\tau_{ep}$ can be expressed as:

$$
\tau_{ep} = -\frac{cK}{ab} \left( \frac{1}{\cosh(l_n)} + \frac{Z_{ep} E_y I_s E_a}{a E_y I_s E_a \cosh(l_n)} \right) M \quad (14)
$$

Additionally, by assuming the condition of $l_n \equiv \infty$

in Eq.(14), $\tau_{ep}$ converges to the following equation.

$$
\tau_{ep} = -\frac{cZ_{ep} E_y I_s E_a}{2aE_y I_s E_a} \varepsilon_{pre} \quad (15)
$$

This equation has no term of the bending moment, $M$. Therefore, by releasing the pre-tension of
CFRP plate with enough length of \( l_n \), the shear stress in adhesive layer just arises at \( x = l_p \).

Comparing the Eq.(14) with Eq.(15), it is found that the function of \( \frac{1}{2} \left[ \cosh(c l_n) + \sinh(c l_n) \right] \) governs the effect of bending moment, \( M \). Replacing \( \frac{1}{2} \left[ \cosh(c l_n) + \sinh(c l_n) \right] \) by \( \lambda \), Eq.(14) becomes following equation.

\[
\tau_{ep} = -\frac{cK}{ab} \left( \lambda M + \frac{1 + \lambda^2}{2} Z_{cl} E I \lambda \right) \tag{16}
\]

Providing enough length of \( l_n \), \( \lambda \) becomes approximately 0. In contrast, giving enormously short length of \( l_n \), \( \lambda \) converges to 1. In the design of length of CFRP plate, therefore, \( l_n \) is decided by substituting quite small value into \( \lambda \). Furthermore, \( \lambda = \frac{1}{2} \left[ \cosh(c l_n) + \sinh(c l_n) \right] \) can be deformed as:

\[
l_n = -\frac{1}{c} \ln(\lambda) \tag{17}
\]

For example, using the values in Table 1 and \( \lambda = 0.01 \), the length of \( l_n \) is calculated as 102mm.

For the conventional CFRP plate bonded steel member, \( \tau_{ep} \) at \( x = l \) without non pre-tensioned region is given by substituting \( \lambda = 1 \) into Eq.(16).

\[
\tau_{ep} = -\frac{cK}{ab} \left( M + \frac{Z_{cl} E I \lambda}{2} \right) \tag{18}
\]

For the comparison of Eqs.(15) and (18), It is found that placing the non-pre-tensioned region can reduce the maximum shear stress induced by releasing the pre-tension up to half of conventional method.

Meanwhile, existence of high shear stress and peel stress at the edge of CFRP plate is well-known. When the length of \( l_n \) is designed by providing \( \lambda \equiv 0 \) into Eq.(17), the shear and peel stresses in adhesive layer converge to following equations, respectively:

\[
\tau_{en} = -\frac{cK}{ab} M \tag{19}
\]

\[
\sigma_{yen} = \frac{2\omega}{bZ_{cl}} \left[ \frac{2}{4 + (c/\omega)^2} - \frac{c}{\omega} \left( \frac{c}{\omega} \right) \right] + \omega(Z_{cl} - 1) M \tag{20}
\]

The stresses by releasing the pre-tension of CFRP plate are not included in these equations. Therefore, by using the proposed method, high shear and peel stresses at the adhesive ends are just induced by bending moment.

Figure 6 shows the shear and peel stresses in adhesive layer obtained by Eqs.(4) and (7). This figure also includes the convergent stresses calculated by Eqs.(15), (19) and (20). The convergent stresses at \( x = l_p \) and at \( x = l \) are corresponding to the shear and peel stress distributions.

3.3 Debonding Bending Moment

The condition for prevention of debonding of CFRP plate is suggested as [3], [4]:

\[
\sigma_s \leq \sigma_p \tag{21}
\]

where, \( \sigma_p \) is the debonding strength of adhesive.
By assigning enough length of $l_n$, the shear and peel stresses at the adhesive ends just depend on the bending moment. Therefore, debonding bending moment of CFRP plate can be determined with the shear and peel stresses at the adhesive ends. As the bending stiffness of CFRP plate is extremely smaller than that of steel member, $Z_s$ is assumed to be 1. Furthermore, $(c/\omega)$ and $(c/\omega)^2$ are regarded as 0. The Eqs.(19) and (20) can be simplified as followings:

\[
\tau_{en} = -\frac{c_1 K_1}{ab} M \tag{22}
\]

\[
\sigma_{pren} = \frac{c_1 \omega_1 d_1 K_1}{ab} \left(2 - \frac{c_1}{\omega_1}\right) M = -\alpha \tau_{en} \tag{23}
\]

where, 
\[
c_1 = \sqrt{\frac{a^2 b G_s}{(bk_s E_s I_s)}}, \quad \alpha = d_1 (2\omega_1 - c_1), \quad \omega_1 = \frac{4b E_s}{(4h E_s I_s)} \quad \text{and} \quad K_1 = 1/\{1 + Z_s (\tau_s/\alpha)^2\}.
\]

Substituting Eqs.(22) and (23) into Eq.(13), the condition for prevention of debonding of CFRP plate can be defined as:

\[
-\frac{\alpha}{2} + \sqrt{\left(-\frac{\alpha}{2}\right)^2 + 1}\tau_{en} \leq \sigma_p \tag{24}
\]

If Eq.(24) is satisfied, debonding of CFRP plate is prevented.

Debonding bending moment is given by substituting Eq.(22) into Eq.(24).

\[
M \leq \frac{2ab}{c_1 K_1 (\alpha + \sqrt{\alpha^2 + 4})} \sigma_p \tag{25}
\]

Meanwhile, shear stress in adhesive layer induced by releasing the pre-tension of CFRP plate concentrates at the pre-tensioned region ends, as mentioned above. Upper bound for providing the pre-tension of CFRP plate is, therefore, determined with $\tau_s$ at $x = l_n$.

\[
e_{pre} \leq \frac{2a^2 b}{c_1 K_1 E_s I_s} \sigma_p \tag{26}
\]

For instance, using the properties in Table 1 and $\sigma_p = 50$ MPa, the limit state conditions for prevention of debonding, which consists of bending moment and providing pre-tension of CFRP plate, are shown in Fig. 7. The limit state condition of conventional CFRP plate bonded onto the steel member, which is given by the following equation [5], is also shown in this figure.

\[
M \leq \frac{2ab}{c_1 K_1 (\alpha + \sqrt{\alpha^2 + 4})} \sigma_p - \frac{E_s I_s}{a} e_{pre} \tag{27}
\]

It is found that the debonding bending moment for conventional method decreases with increasing of pre-tension provided. However, the applicable limit of bending moment in the proposed method can be dramatically improved because the debonding bending moment is independent from the effect by releasing the pre-tension of CFRP plate.

4. CONCLUSIONS

In this study, to improve the debonding bending moment of pre-tensioned CFRP plate bonded onto steel members, installing the non pre-tensioned regions at the CFRP plate ends was proposed. Furthermore, to verify whether the proposed method is effective, theoretical investigation was carried out under the simple loading condition of pure bending. Main conclusions are as follows:

1. By installing the non pre-tensioned region at the ends of CFRP plate, the maximum shear stress in adhesive layer induced by releasing pre-tension of CFRP plate appears at the pre-tensioned region ends. Additionally, the maximum shear stress of proposed method is reduced up to half of that of conventional one.

2. By installing the enough non pre-tensioned region, the applicable limit of bending moment and pre-tension for prevention of debonding of CFRP plate was improved compared with conventional pre-tension induced method.

In the further research, more practical loading condition should be confirmed. Experimental tests also should be conducted to verify the effectiveness of proposed method.

APPENDIX

The matrix corresponding to boundary condition and the vector corresponding to applied moment and provided pre-tension are described as Eqs.(A1) and (A2). In the Eq.(A1), $CH(*)$, $SH(*)$, $C(*)$, $S(*)$ mean $\cosh(*)$, $\sinh(*)$, $\cos(*)$, $\sin(*)$, respectively.
REFERENCES


