Geometrically Nonlinear Stress Analysis for Imperfect CFRP Reinforced Steel Cylinders under Compression

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ABSTRACT

Steel cylindrical shells when exposed to the hostile chemical on marine environments show their vulnerability to corrosion. At the same time, it becomes difficult to achieve required level of load carrying capacity by using shells constructed just from fibre reinforced polymer. In this context, carbon fiber reinforced polymer (CFRP) if collectively used with thin steel shells, the physical and chemical properties as a new composite structure will increase tremendously. On our previous study (CICE 2010 in Beijing), it has been predicted that the buckling load carrying capacities of thin cylindrical shells increase with CFRP reinforcement through the linear buckling, the reduced stiffness (RS) and the nonlinear imperfection analyses. The present extensive research study discusses the way of an alternative simple confirmation of elastic nonlinear stress variations near the interface between CFRP and steel layers in any prebuckling equilibrium states.

KEYWORD

Carbon fibre reinforced polymer(CFRP) , Linear buckling analysis, Reduced stiffness analysis (RS analysis), Nonlinear imperfection analysis, Buckling strength, Composite structures, Bond stress, Cylindrical shell
1. INTRODUCTION

The constant demand and necessity for lightweight efficient structures continue to encourage engineers to the field of structural optimization and simultaneously to the use of non-conventional materials. The strength properties of fibre reinforced polymeric (FRP) matrix composites make one of the ultimate reason for which civil engineers prefer them in the design of structures. In many situations, FRP composites provide opportunities for enhanced efficiency because of their high strength-to-weight ratios. Also, researches have shown that these FRP’s are ideally suited for retrofitting purpose and it has great potentiality to upgrade the strength properties of steel structures. That’s why; these composites are particularly suitable for the design of bridges, large span structural members, aerospace components and pressure vessels. Since steel shells are considerably stiffer than the CFRP composites, strengthening them requires expensive high-strength fibres and, thus, this procedure has been generally deemed not advantageous. Despite this fact, El Damatty et al. (1) have shown both experimentally and numerically, that glass fibre (GFRP) sheets can be used to enhance the load-carrying capacity. Nevertheless, there is no doubt that CFRP are, of course, expensive and less processable than GFRP, but has predominant advantage of high stiffness. However, these composites have drawback having relatively lower stiffness driven by CFRP’s. Consequently, serviceability rather than strength limit states tend to provide the controlling influence on design constraint in the context of thin-walled shell structures. So that the required buckling strength couldn’t be obtained for the shells constructed from CFRP only. In this case, a novel way to improve this drawback of CFRP would be, jointly use with thin-walled steel shells. So that steel with high yield stress could be strengthened and possibility of corrosion inside the marine environments also would be vanished because the carbon fibres are chemically inert and have low surface energy. These properties result in low wettability of these fibres with solutions and melts of typical materials and, as a result, low adhesions exist at the surface of fibre-matrix interface. Also, it is considered that steel CFRP lamina is fabricated using perfect adhesives to treat as a composite structure, which also helps to reduce the possibility of galvanic corrosion. In this study, CFRP coated thin-walled steels shells have been studied through three kinds of analytical procedure; the conventional linear buckling analysis, the reduced stiffness (RS) buckling analysis and the nonlinear imperfection analysis. These multiple treatments suggest obtaining valuable information for the design of CFRP based hybrid structural elements and discusses influence of CFRP to increase the load carrying capacity of the thin-walled metallic structures having complex buckling collapse behaviour, CICE 2010 (2). Since the mechanical behaviour of CFRP shells is much dependent upon the fibre orientation Matsumoto et al. (3), the relative fibre orientation in the case of $\theta = 0^\circ$ and $\theta = 90^\circ$ has been given priority for the analysis. It is well known that axially compressed cylindrical shells have a buckling behaviour which is very sensitive to initial geometric imperfections (4). In the case of CFRP material, the angles and dispositions of fibre orientations, as well as the magnitudes of any imperfections, have been suggested to affect the buckling behaviour (3, 5). To predict the lower bound for the larger imperfection sensitive buckling loads, a modified theory has been developed in this study and suggested that this modified RS theory has the potentiality to provide simple and reliable estimation for designing the larger imperfection sensitive buckling loads for CFRP reinforced steel cylinders under axial compression. Also, for the actual application of this sandwich structural system, it would be needed to check the nonlinear stress variation near the interface between CFRP and steel layers in any equilibrium states. The present paper will discuss on how to obtain it through step-by-step calculation procedure.

2. METHOD OF ANALYSIS

2.1. CFRP Lamina and CFRP Reinforced Steel Lamination

As shown in Fig. 1 a section of thin-walled CFRP reinforced steel cylinder is considered in which $x$-$y$ denotes the coordinate of thin cylindrical shell and 1-2 denotes the coordinates along fibre direction. The material constants are obtained by using Halpin-Tsai equation (6) as

$$\begin{align*}
E_1 &= E_F V_F + E_P V_P, \\
\mu_{21} &= \mu_F V_F + \mu_P V_P, \\
\mu_{22} &= \frac{E_2}{E_1} \mu_{12}
\end{align*}$$

(1)

In Eq.1 subscript $F$ and $P$ relate to fibre and polymer, respectively. $E_1$ represent elastic
coefficient and $E_F$ and $E_p$ as elastic constants for fibre and polymer, respectively. Also, $V_F$ and $V_p$ represent volume fraction for fibre and polymer and $\mu_2$ and $\mu_1$ as Poisson’s ratios. In Eq.1 $E_2$ is the elastic coefficient normal to the fibre and calculated as

$$E_2 = E_p \left(1 + \xi \eta V_p \right) / \left(1 - \eta V_p \right)$$

Parameters $\xi$ is taken as $= 2$ and $\eta = \left((E_F / E_p) - 1\right) / \left((E_F / E_p) + \xi \right)$ . Again, the shear modulus of elasticity $G_{12}$ is calculated as

$$G_{12} = G_p \left(1 + \xi \eta V_p \right) / \left(1 - \eta V_p \right)$$

and the associated parameter $\xi$ for the calculation of $G_{12}$ is taken as $\xi = 1 + 40 V_p^0$ and $\eta$ for the calculation of $G_{12}$ is $\eta = \left((G_F / G_p) - 1\right) / \left((G_F / G_p) + \xi \right)$.

The resulting transformed linear elastic constants after the transformation of the linear elastic constants from the principal material fibre directions to a global $x$-$y$ coordinate is as below

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & 0 \\ 0 & 0 & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} + \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

(2)

Where,

$$Q_{11} \equiv E_1 / (1 - \mu_2 \mu_3), Q_{12} \equiv \mu_2 E_1 / (1 - \mu_2 \mu_3), Q_{22} \equiv E_2 / (1 - \mu_2 \mu_3)$$

and $\left(\varepsilon_x, \varepsilon_y, \varepsilon_{xy}\right)$ are the principal stress and strain components associated with $x$-$y$ plane, and similarly, $\left(\varepsilon_x, \varepsilon_y, \varepsilon_{xy}\right)$ and $\left(\kappa_x, \kappa_y, \kappa_{xy}\right)$ are the corresponding membrane and bending strains on the middle plane of the shell respectively. Also, by integrating the whole thickness of lamina, we can obtain the membrane and bending stress resultant matrices as

$$\begin{bmatrix} n_x \\ n_y \\ n_{xy} \end{bmatrix} = \begin{bmatrix} A_{y} \\ A_{x} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} + \begin{bmatrix} B_{y} \\ B_{x} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \end{bmatrix} + \begin{bmatrix} D_{y} \\ D_{x} \end{bmatrix} \begin{bmatrix} \kappa_{xy} \end{bmatrix}$$

(3)

Where, $(n_x, n_y, n_{xy})$ and $(m_x, m_y, m_{xy})$ are the total membrane and bending stress resultants respectively. Similarly, $A_{y}, B_{y}$ and $D_{y}$ are respectively the membrane, membrane bending coupling and bending stiffness respectively.

From Eq. (3) the constitutive relation for the laminated plate can be calculated as

$$\begin{bmatrix} m_x \\ m_y \\ m_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} n_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} n_{xy} \end{bmatrix}$$

(4)

In the present study, symmetric laminations are adopted. So that, all the components of $B_{y}$ will be zero.

3. NONLINEAR IMPERFECTION ANALYSIS

For an imperfect CFRP reinforced thin-walled steel cylinders the change in the total potential energy, consequent upon the application of axial compression $P$ may be written as

$$\Pi = U_m + U_b + V$$

(5)

where $U_m$ are the membrane strain energies, $U_b$ are the bending energies and $V$ the increase in load potential.

$$U_m = \frac{1}{2} \int_0^{L} \int_0^{2\pi R} (n_x \varepsilon_x + n_y \varepsilon_y + 2n_{xy} \varepsilon_{xy}) \, dx \, dy$$

(6)

$$U_b = \frac{1}{2} \int_0^{L} \int_0^{2\pi R} (m_x \kappa_x + m_y \kappa_y + 2m_{xy} \kappa_{xy}) \, dx \, dy$$

$$V = -\frac{P}{2\pi R} \int_0^{2\pi R} \int_0^L \left( -\frac{\partial u}{\partial x} \right) \, dx \, dy$$

To get the strain-displacement relationship Donnel-Mushtari-Vlasov type is adopted for the deformations from the initial imperfections $w^0$ as

$$\begin{align*}
\kappa_x &= -\frac{\partial^2 w}{\partial x^2}, & \kappa_y &= -\frac{\partial^2 w}{\partial y^2}, & \kappa_{xy} &= -\frac{\partial^2 w}{\partial x \partial y} \\
\varepsilon_x &= \frac{\partial u}{\partial x} + w + \frac{1}{R} (\frac{\partial w}{\partial y})^2, \\
\varepsilon_y &= \frac{1}{R} \frac{\partial u}{\partial y} + w + \frac{1}{R} (\frac{\partial w}{\partial x})^2.
\end{align*}$$
\[ \varepsilon_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} + \frac{\partial w^0}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial w^0}{\partial y} \frac{\partial w}{\partial x} - \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) \]  

(7)

End boundaries are assumed to be supported in such a way as to conform to the classical simple support, corresponding with the conditional expression as

\[ w = 0, \partial^2 w / \partial x^2 = 0, \partial u / \partial x = 0, \nu = 0 \text{ at } x = 0, L \]  

(8)

The linear sum of bi-harmonic function that satisfy the above boundary condition, the displacement functions \(u\), \(v\) and \(w\) as

\[ u = \sum_{i=0}^{b} \sum_{j=1}^{2} u_{ij} \cos \left( \frac{iy}{R} \right) \cos \left( \frac{j\pi x}{L} \right) \]  

\[ v = \sum_{i=0}^{b} \sum_{j=1}^{2} v_{ij} \sin \left( \frac{iy}{R} \right) \sin \left( \frac{j\pi x}{L} \right) \]  

\[ w = \sum_{i=0}^{b} \sum_{j=1}^{2} w_{ij} \cos \left( \frac{iy}{R} \right) \sin \left( \frac{j\pi x}{L} \right) \]  

where, \(u_{ij}\), \(v_{ij}\) and \(w_{ij}\) are the amplitudes of each harmonic function; \(i\) and \(j\) are the circumferential full-wave and the longitudinal half-wave number, respectively. The initial geometric imperfection is taken to consist of a harmonic of

\[ w^0 = w_{0f} \cos \left( \frac{bfy}{R} \right) \sin \left( \frac{f\pi x}{L} \right) \]  

(10)

in which \(b\) and \(f\) represent the circumferential full-wave and longitudinal half-wave number, respectively.

3.1 Analytical Prediction for the Stress at the Interface of CFRP and Lamina

As shown in the Fig. 3 below, \( \bar{\varepsilon}_{xx}, \bar{\varepsilon}_{yy}, \bar{\varepsilon}_{xy} \) represents the strain inside the membrane.

\[ \bar{\varepsilon}_{xx}, \bar{\varepsilon}_{yy}, \bar{\varepsilon}_{xy} \]

CFRP (F)

Steel (S)

CFRP (F)

Fig. 2: Lamination of steel and CFRP lamina for FFS model

Normalizing Eqs. 9 and 10, respective displacement for each part is established adopting the parameter

\[ \xi = \frac{x}{L}, \eta = \frac{y}{2\pi R} \]

where, \(x = L/2, y = 0\). Consequently, we can obtain the strain with varying distance \(z\) as below,

\[ \bar{\varepsilon}_{xx} = \varepsilon_x + 2\xi, \bar{\varepsilon}_{yy} = \varepsilon_y + 2\eta, \bar{\varepsilon}_{xy} = \varepsilon_{xy} + \xi \eta \]

\[ \begin{align*}
\bar{\varepsilon}_{xx} &= \varepsilon_x + 2\xi
\quad = \frac{\partial u}{\partial x} + \frac{\partial w^0}{\partial x} \frac{\partial w}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 - z \frac{\partial^2 w}{\partial x^2} \\
\bar{\varepsilon}_{yy} &= \varepsilon_y + 2\eta
\quad = \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{\partial w^0}{\partial y} \frac{\partial w}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - z \frac{\partial^2 w}{\partial y^2} \\
\bar{\varepsilon}_{xy} &= \varepsilon_{xy} + \xi \eta
\quad = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w^0}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial w^0}{\partial y} \frac{\partial w}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) - z \frac{\partial^2 w}{\partial x \partial y} 
\end{align*} \]  

(11)

Consequently,
4. RESULTS AND DISCUSSION

For the analysis of reinforced shells, steel and FRP’s are laminated with constant steel with wall thickness of $t_s = 4$mm as shown in Fig.2 and the adopted geometrical parameters are $L/R = 0.512$ and $R/t_s = 405$. Also, $t_f$ represents the thickness of carbon fibre ranging the thickness of fibre from 0 to $t_f$. While, Young’s moduli for steel, fibre and polymer are taken as $E_s = 205$GPa, $E_f = 235$GPa, $E_p = 3.5$GPa, and Poisson’s ratios for steel, fibre and polymer are $\mu_s = 0.3, \mu_f = 0.3$ and $\mu_p = 0.34$, respectively.

Fig. 3 shows the results of numerically experimented load-deflection curves for $b=10$ having imperfection amplitudes of 0.8, 2.4, 5.6, 6.4 and 7.8mm and shown that the buckling load carrying capacity is the highest for the reinforced condition having $\theta=90^\circ$ for all the amplitudes. Fig. 4 shows the incremental buckling displacements at the buckling loads for the cases of an imperfection amplitude $w_{yi0} = 7.8$mm. From this schematic figure, it can be observed that the axial wave length becomes sharper for the cases of reinforced conditions; this modified mode form reflects the influence of the CFRP reinforcement on the present numerical experimental models. Again, Fig. 5 shows the typical significant changes in mode at buckling as compared with the form of initial imperfection in the case of axial ($\gamma=0$) and circumferential ($\gamma=L/2$). In Fig.6 an imperfection with $b=10$ is adopted since this mode results in the minimum nonlinear buckling loads. Whereas, the linearised critical buckling loads, and the nonlinear buckling loads for very small imperfections, exhibit considerable dependence upon the angle of fibre orientation, the buckling loads for large...
imperfections show remarkably load dependence upon the angle of fibre orientation.

Fig. 7 is based upon the use of larger imperfections having a form \((h, f) = (10, 1)\) and are observed to produce buckling loads that are lower than \(P^*_{cm}\) associated with the mode \((\iota_{cm}, 1)\). But what is fascinating about the nonlinear results is that despite the shape of initial imperfection, \((w^0_{0,i})\), the incremental mode at buckling, at least when imperfection amplitudes are large, is dominated by wave form having considerably shortened circumferential and axial wave lengths. For the case of \(t_f = 2\)mm and \(\theta = 90^\circ\) shown in Fig. 5, for example, the incremental mode at buckling for the large imperfection \(w^0_{0,1} = 7.8\)mm, has through a process of modal coupling reached localised shapes closer to that associated with \((i,j) = (10, 2.26)\). The lowest RS critical load associated with larger imperfection is taken as modified RS load \(P^*_{cm}\) in this paper and it is no coincidence that this mode also happens to be the same as that for the lowest RS critical load associated with \(\iota_{cm}\) should then be that of \(P^{*}\) for \((\iota_{cm}, 2.26)\), which is depicted as \(P^*_{cm} = 3.19\)MN in Fig. 7. Making use of this modified RS critical load (i.e. the lowest \(P^{*}\) associated with the mode \((\iota_{cm}, 1)\)) can be seen to consistently provide extremely close approximations of the lower bounds for larger imperfection sensitive buckling loads.

Figs. 8a and 8b represent the relationship between buckling load and the thickness of fibre \(t_f\) for angle of fibre orientation. Since, \(P^*_{cm} < P_f\) by modified RS analysis suggests that the elastic buckling for moderately large imperfect shells in the use of practical civil engineering structures will occur first rather than material damage inducing collapse, \(P^*_{cm}\) provides consistent and reliable lower bounds over the entire range CFRP reinforcements considered. In Fig. 9, the linear buckling load \(P_m\) becomes optimum at an angle of fibre orientation \(35^\circ\). But as the results of nonlinear numerical experiments show,

\[
\begin{align*}
\sum \sum w_i^j \sin(\xi x / L) & \\
\sum \sum w_i^j \sin(\xi x / L) & \\
\sum \sum w_i^j \sin(\xi x / L) & \\
d: \sum \sum w_i^j \cos(\eta y / R)\sin(\eta \pi / 2) & \\
e: \sum \sum w_i^j \cos(10\eta y / R)\sin(\eta \pi / 2) & \\
f: \sum \sum w_i^j \cos(20\eta y / R)\sin(\eta \pi / 2) & \\
\end{align*}
\]

with angles of fibre orientation \(20^\circ, 35^\circ\) and \(70^\circ\), the imperfect shell buckling loads \((P^{nv})\) and the RS critical loads are approximately the same for the imperfection amplitude \(w^0_{0,1} = 2 \times t_f\).
5 CONCLUSIONS

In this paper an alternative simple procedure on the confirmation of elastic nonlinear stress variations near the interface between CFRP and steel layers has been proposed. In additions, it have been shown that new modified reduced stiffness criterion provides the reliable estimation for designing the larger imperfection sensitive buckling loads predicting the lower bound for CFRP reinforced steel cylinders under axial compression.

REFERENCES


