Prediction of the Flexural Behavior of Fibre Composite Sandwich Beams

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ABSTRACT

One of the core assumptions underlying the construction industry is that the true potential of fibre composite sandwich structures can only be exploited through the use of complex mathematical theories and sophisticated analysis techniques. Research experience at the Centre of Excellence in Engineered Fibre Composites at the University of Southern Queensland has shown that with correct material data, the behaviour of composite sandwich structures can be analysed within acceptable levels of confidence using simple techniques. In this paper, a simplified Fibre Model Analysis (FMA) to describe the approximate behavior and the governing failure mechanisms of composite sandwich structures under flexural load is presented. This fundamental design methodology is based on sectional equilibrium, strain compatibility, and the constitutive material behaviour using a layer-by-layer approach to evaluate the sectional forces and to calculate the nominal flexural capacity of a composite sandwich section. An important advantage of the proposed model is that it can account for the nonlinear behaviour of the core in compression, the effect of core cracking in tension and the linear elastic behaviour of the fibre composite skins. This method also allows analysing composite sandwich structures with non-symmetric section and using only MS Excel spreadsheet program. The efficiency and practical application of this prediction equation is demonstrated by analysing the behavior of individual and glue-laminated composite sandwich beam structures made from the glass fibre composite skins and phenolic core material and comparing with the results of experimental investigation and numerical simulation. This method was found to reasonably describe the behavior of the fibre composite sandwich structures in flexure, therefore suggested as very useful for engineering design and calculations.

KEYWORD

Sandwich structures, fibre composites, Fibre Model Analysis, design methods, flexure
1. INTRODUCTION

The many advantages of composite sandwich structures have drawn a lot of attention in the construction industry and for civil engineering applications [1]. The benefits of sandwich construction are their performance, and better bending strength and stiffness [2]. These materials are now commonly used as structural panel for roofs, floors, walls, and bridge decks. However, very limited attempt has been made so far to use these materials for structural beam applications. The main reason for this could be that most of the currently used core are not appropriate for this type of application. Foam core and balsa wood are soft and will crush under high compressive loads. Honeycomb and trussed-core structures have high compressive strength but the presence of cavities in these core materials reduces their capacity to hold mechanical connections. The evolution of a sandwich structure with lightweight, high strength core and with good holding capacity for mechanical connections provides an opportunity to develop this material for structural beams.

A new generation fibre composite sandwich panel made up of glass fibre-reinforced polymer skins and modified phenolic core material (Figure 1) has been developed in Australia [3]. The improvement in the structural performance of this new generation sandwich structure presents an ideal opportunity to increase the use of this material for civil infrastructure. An innovative structural beam concept made from this sandwich panel has been developed by the authors [4]. The innovative beam concept was made by gluing a number of these sandwich panels together. The structural behaviour of these beams has shown that gluing these panels together resulted in a more stable and stronger section than individual sandwich structures. However, before this system can be used effectively for civil infrastructure and in various engineering applications, a simple prediction equation that can capture many of the important variables that influence the flexural behavior of fibre sandwich structures should be developed.

In the analysis of sandwich structures it is usually assumed that the core only supports the shear and the skins carry the tensile and compressive loads under flexure [5]. In the structural sandwich beams, the contribution of the core and the skin in both flexural and shear stiffness were considered. However, the simplified mechanics equation for the calculation of bending stiffness for symmetrical composite sandwich section cannot be used directly for sandwich structure with high strength core material as the elastic properties of the constituent materials is different in tension and compression. An analysis method to allow the cracking of the core, commonly used in reinforced concrete, should be incorporated in the analysis of the flexural behaviour of the sandwich beams.

![Fig.1 The novel composite sandwich panel](image)

The behaviour of fibre composite sandwich beams in flexure was predicted using a Fibre Model Analysis. This fundamental design methodology is commonly used in the analysis of the conventional steel-reinforced concrete and FRP-reinforced concrete. According to the ACI 440R-2007 [6], regardless of the reinforcing material, the design of reinforced concrete structures is based on the cross-sectional equilibrium, strain compatibility, and the constitutive material behaviour. This method has been used successfully by a number of researchers in the analysis of the behaviour of concrete structures reinforced with fibre reinforced polymer (FRP) materials. Duthinh and Starnes [7] presented an iterative numerical approach to predict the flexural strength enhancement of FRP strip bonding to steel reinforced concrete beams. Their approach involved iteration of two variables – the depth of compressive block and concrete compressive strain at extreme fibres. Similarly, El-Hacha et al. [8] analyse the flexural behaviour of a concrete beam strengthened with near-surface mounted FRP reinforcement using simple plane section analysis.

This paper presents the results of theoretical evaluation on the flexural behaviour of a fibre composite sandwich beam structure with a high strength core material using the FMA. The accuracy of this analysis method was determined by comparing the predicted results with the experimental [4] and numerical investigations.
2. MATERIAL CONSTITUTIVE MODELS

The constitutive model of the skin and the core of the composite sandwich structure were determined through physical testing using coupon specimens following the ISO and ASTM standards.

2.1 Fibre composite skin

The top and bottom skins of the innovative composite sandwich structure is made up of 2 plies of stitched bi-axial (0/90) E-CR glass fibre fabrics with a toughened phenol formaldehyde resin for the laminate matrix [3]. The skin has a fibre fraction of 45% by weight and an average density of 1365 kg/m$^3$.

In the longitudinal direction, the fibre composite skin is modelled as a linear elastic material in both tension and compression with a slightly higher modulus in compression (16,100 MPa compared to 15,380 MPa) but has a higher strength in tension. The fibre composite skin will fail in tension when the strength $f_{(t)}$ and strain $\varepsilon_{(t)}$ reach 246 MPa and 0.016, respectively, while the failure of the skin in compression will occur when the compressive strength $f_{(c)}$ of 201 MPa and strain $\varepsilon_{(c)}$ of 0.0124 are reached.

2.2 Modified phenolic core

The phenolic core material is a proprietary formulation by LOC Composites Pty Ltd, Australia. This material comes from natural plant products derived from vegetable oils and plant extracts and chemically bonded within the polymer resin. It has an average density of 855 kg/m$^3$. The modified phenolic core material behaved linear elastic (up to failure) in tensile but showed a non-linear elastic behaviour in compression. Also, its average compressive strength of around 21 MPa is comparable to that of normal density concrete. Furthermore, the tensile strength of phenolic core is almost 25% of its compressive strength. This almost similar behaviour of the phenolic core material to concrete was therefore accounted in the analysis.

A linear stress-strain relation is used to model the phenolic core in tension while a simplified bilinear stress-strain behaviour is used in compression. The bilinear stress-strain behaviour of the core in compression is characterised by linear elastic up part to a limiting strain of 0.016 then a constant stress of 21.35 MPa until the failure strain of 0.035.

3. PREDICTION OF SANDWICH BEHAVIOR

3.1 Fibre Model Analysis (FMA)

The FMA model is based on the layer-by-layer approach to evaluate the sectional forces corresponding to a given strain distribution at a specific section [9]. The nominal flexural capacity was calculated from the constitutive behaviours of the skin and the core using strain compatibility and internal force equilibrium principles. Due to the high strength of the phenolic core, a perfect bond between the skins and the core was assumed, and the strains in the skins and the core were assumed directly proportional to their distance from the neutral axis. These assumptions were based on Bernoulli’s hypothesis of strain compatibility: that plane sections remain plane which require perfect bonding between the skins and the core material, and that no slip occurs. Similarly, a perfect bond is assumed between adjacent layers of the skins. The stress was computed by multiplying the strain to the modulus of elasticity of the materials. The internal force at each layer was calculated by multiplying the stress to the area of layer and the cross-sectional force equilibrium (in summation of forces, the net tensile force shall be equal to the net compressive force) was applied. MS Excel spreadsheet program was used to analysed the flexural behavior of the beams using the FMA.

The basic assumptions in FMA are illustrated in Figure 2. When the core is still uncracked, all the layers or element $i$ (with thickness, $t_i$) contribute to the moment capacity of the section as illustrated in Figure 2a. However, the contribution of the core in tension is neglected when the core cracks (Figure 2b). In addition, the average thickness and in-plane shear properties obtained experimentally of 2470 MPa and 530 MPa for the skin and core, respectively are used in the analysis. In the figure, $f_{(t)}$ and $\varepsilon_{(t)}$ represent the cracking strength in tension and the corresponding cracking strain of the core material respectively while $f_{(c)}$ and $\varepsilon_{(c)}$ are the compressive strength and the corresponding elastic strain, respectively.

The initial iteration starts by assuming a compressive strain value at the topmost layer of the fibre composite sandwich structure. For a given top strain, the bottom strain is solved for by iteration until the summation of forces is equal to zero. The corresponding neutral axis depth for these set of top and bottom strains which satisfies the force equilibrium principle is calculated using
the equation (1). The strain, $\varepsilon_i$ at element $i$ is related to the top strain, distance from the top of the element, $y_i$ and the curvature, $\varphi$ of the composite sandwich section which can be determined using equation (2). Based on Figure 4, the curvature of the composite sandwich section or the rotation per unit length of the beam can be determined using the equation (3). The stresses are then calculated from the strain at each layer multiplied by its corresponding elastic modulus.

\[
y = \frac{-\varepsilon_{s(c)} D}{(\varepsilon_{s(t)} - \varepsilon_{s(c)})}
\]

(1)

\[
\varepsilon_i = \varepsilon_{s(c)} - \varphi y_i
\]

(2)

\[
\varphi = \frac{\varepsilon_{s(c)} + \varepsilon_{s(t)}}{D}
\]

(3)

where $\varepsilon_{s(c)}$ is the strain in the extreme fibre composite skin in compression and $\varepsilon_{s(t)}$ is the strain in the extreme fibre in tension. The force equilibrium equations and nominal flexural capacity can be written as equations (4) and (5), respectively. In these equations, $n$ corresponds to the number of layers that the composite sandwich section was subdivided, $f_{s(c)i}$, $f_{c(c)i}$, $f_{c(t)i}$, and $f_{s(t)i}$ are the stresses at each layer of the skin in compression, core in compression, core in tension and skin in tension, respectively while $A_{s(c)i}$, $A_{c(c)i}$, $A_{c(t)i}$, and $A_{s(t)i}$ are the corresponding areas of each layer of the skin in compression, core in compression, core in tension and skin in tension, respectively and $M_n$ is the nominal flexural capacity of the composite sandwich section.

\[
\sum P = \sum_{i=1}^{n} f_{s(c)i} A_{s(c)i} + \sum_{i=1}^{n} f_{c(c)i} A_{c(c)i} + \sum_{i=1}^{n} f_{c(t)i} A_{c(t)i} + \sum_{i=1}^{n} f_{s(t)i} A_{s(t)i} = 0
\]

(4)

\[
M_n = \sum_{i=1}^{n} f_{s(c)i} A_{s(c)i} y_i + \sum_{i=1}^{n} f_{c(c)i} A_{c(c)i} y_i + \sum_{i=1}^{n} f_{c(t)i} A_{c(t)i} y_i + \sum_{i=1}^{n} f_{s(t)i} A_{s(t)i} y_i
\]

(5)
3.2 Failure load
The maximum load that the sandwich section can carry was determined using the simplified FMA. In the experimental test in [4], it was observed that the failure of composite sandwich beams under static bending is initiated by the compressive failure of the topmost skin. As indicated in section 2.1, the compressive failure of the skin occurs when the top strain reaches 12400 microstrains. In the calculation of the failure load, the contribution of the core in tension was neglected when the tensile strength of the core is reached.

3.3 Flexural stiffness, EI
The equivalent flexural stiffness of a sandwich section is the sum of the flexural stiffness of the different parts in the sandwich structure. However, the simplified equation for the calculation of EI for symmetrical sandwich section cannot be used directly for the structural composite sandwich beam as the elastic properties of the constituent materials are different in tension and compression. Similarly, both the contribution of the skin and the core were considered in the calculation of EI. This is also the case for sandwich structures with carbon/epoxy skins and Divinycell core studied by Gdoutus and Daniel [10], wherein the stress-strain behaviour of the skin is different in tension and compression. This consideration is however in contrast with the assumption made by Daniel and Abot [11] and several other researchers where the core contribution is neglected in the calculation of the flexural stiffness of the sandwich structures. The relatively higher compressive modulus of the core and skin than tensile modulus suggested that the neutral axis is not lying at the mid-depth of the section even though the composite sandwich structure was fabricated with symmetrical top and bottom skins. As discussed in the earlier section, the neutral axis depth can be calculated from the set of top and bottom strains that satisfies the force equilibrium principle in equation 4. The EI of the composite sandwich beams can now be estimated using equation (6) while the shear stiffness GA to account for shear deformation can be calculated using equation (7). These relationships are used further for the EI and GA of the composite sandwich beams with cracked core material.

\[ EI = \sum_{i=1}^{n} \left( \frac{Bt_i^3}{12} + Bt_id_i^2 \right) E_i \] (6)

\[ GA = \sum_{i=1}^{n} Bt_i G_i \] (7)

3.3 Load-deflection behaviour
The load-deflection behaviour of the composite sandwich structures was obtained using the shear deformation theory proposed by Timoshenko [12]. In the Timoshenko beam theory, the total deflection is the sum of the deflections due to bending and shear deformations. The relatively low shear stiffness of the core compared with that of the skin usually results in a significant shear deformation that should be accounted in the total deflection of sandwich structures [13]. The maximum mid-span deflection \( \Delta \) for a simply supported composite sandwich beam under 4-point bending with the load applied at 0.4 and 0.6 of the span, \( L \) is given by (8):

\[ \Delta = \frac{59PL^3}{300(El)} + \frac{PL}{5kGA} \] (8)

A shear correction factor, \( k = 1.0 \) is assumed in the analysis. As long as the core remains uncracked, the flexural and shear stiffness of the sandwich beam are equal to the sum of the EI and GA of all layers taken about the neutral axis depth of the uncracked section. When tensile strength of the core is reached, the EI is calculated as the sum of the EI of all the uncracked layers while the skin and core layers under the cracked core were neglected in the calculation of GA. Similarly, the assumptions made by Natterer and Hoeft [14] on the prediction of the load and deflection behaviour of hybrid timber-concrete girders that only a single cross-section over the whole length of the beam is considered.

4. FE ANALYSIS OF SANDWICH BEAM BEHAVIOUR

Numerical simulations using finite element (FE) analysis were carried out to verify the accuracy of the simplified FMA and to compare with the experimental measurements in [4]. The simulation of the behavior of sandwich structures under 4-point static bending test was performed using Strand7 finite element program [15] in the FCD-XPP-034 computer (CPU-Intel P4). The FE analysis was carried out simulating the specimen and the loading set-up in the actual experimental conditions to have a reliable result. Due to symmetry, only one-fourth of the sandwich beams was modelled to reduce the computational time. The skin and the core were modelled as 20-node hexahedron (Hexa20) solid/brick elements with aspect ratios between 1.1 and 1.4. Figures 3 and 4 show the numerical model used to simulate the beams with 1 and 4 laminations, respectively.
In the FE simulation, nonlinear static analyses were conducted considering the behaviour of the constituent materials as described in sections 2.1 and 2.2. The fibre composite skin was assumed to be perfectly bonded to the core, eliminating the debonding failure mode. Analysis was conducted using the nonlinear static solver function in Strand7. The failure of the sandwich beam is defined in the FE analysis as the strain at which the maximum strength in the elements exceeded either the maximum tensile, compressive or shear strength of the material. After each analysis, the deflection, bending stress-strain relationships at the topmost and bottommost brick elements at the midspan of the composite sandwich beams and the shear stress in the phenolic core at each load increment are recorded.

5. PREDICTED RESULTS AND COMPARISON WITH EXPERIMENTS

The results of the analytical prediction and numerical simulations of the flexural behaviour of the sandwich beams and comparison with the experimental results in [4] are discussed here.

5.1 Failure load and mechanisms
Table 5 summarises the predicted load at first core crack and failure load of the fibre composite sandwich beams based on FMA and FEM simulations and the actual loads at first core crack and at failure based on experimental investigations. In the table, specimens 1LSW, 2LSW, 3LSW, and 4LSW corresponds to glued beams with 1, 2, 3, and 4 sandwich laminations respectively. The cross-sectional dimension and test span of these sandwich beams are provided in details in [4].

Table 5 Actual and predicted failure load in flexure

<table>
<thead>
<tr>
<th>Name</th>
<th>First core crack (N)</th>
<th>Failure load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual FMA FEM Actual FMA FEM</td>
<td></td>
</tr>
<tr>
<td>1LSW</td>
<td>1604 1780 1800 4554</td>
<td>4406 4500</td>
</tr>
<tr>
<td>2LSW</td>
<td>4387 4833 4800 9472</td>
<td>10915 9200</td>
</tr>
<tr>
<td>3LSW</td>
<td>4188 4027 4100 9318</td>
<td>8940 9100</td>
</tr>
<tr>
<td>4LSW</td>
<td>9186 8967 9100 20869</td>
<td>19710 19800</td>
</tr>
</tbody>
</table>

Using the maximum strain of 6000 microstrains when the core cracks in tension, the FMA and the FEM simulations were successful in the prediction of the load at first crack of the composite sandwich beams. The difference between the predicted and the actual load is only 3 to 10%. The slightly higher load measured at the first cracking of the core compared to the results of the analysis could be associated with the physical constraints of the experiment or could be due to the variation of the composite sandwich beam dimensions. In terms of ultimate load at failure, the predicted failure load of the sandwich beams is almost equal to that of the actual failure load. The predicted failure load due to the compressive failure of the skin (at a maximum strain of 12400 microstrains at the topmost skin) is nearly equal to that of the actual failure load, except for specimen 2LSW-F which failed due to the combined effect of flexure and shear. This failure load due to combined flexure and shear was reasonably predicted in the FEM simulation. In general, the difference between the predicted and the actual load is only 4-5%. This shows that the simplified FMA gave a conservative but reasonable value for the failure load of the glued composite sandwich beams.

5.2 Load-deflection behaviour
The comparison of the experimental and analytical load-deflection behaviour of the glue-laminated composite sandwich beams are shown in Figures 5 to 8. In these figures, the predicted load-deflection behaviour of glue-laminated sandwich beam using FMA and FEM simulations and the results of the experimental investigation are designated with FMA, FEM and Expt, respectively. The figures show that the predicted load-deflection relation based on a simple FMA and FEM simulations is in good agreement with the experimental results especially before cracking in the core material. In all the specimens tested, the load increases with deflection until cracks in the core. A decrease in stiffness was observed after this load which was represented by a small kink in the load-deflection relation curve.
A slight disparity between the predicted load-deflection behaviour and the results of the experiment was observed after cracking of the core. This difference in the load-deflection behaviour after core cracking can be due to the initiation of debonding between the skin and the core which reduces the stiffness of the beam. This complex behaviour could not be predicted using the simplified FMA and even with the FEM simulations. Nonetheless, the difference in the predicted and experimental results is only 4%.

5.3 Load and longitudinal strain relationship
The comparison of the predicted load and longitudinal strain relationship determined using FMA, FE analysis and the experimental result is shown in Figure 9. In this figure, the longitudinal tensile strain is designated with (T) and the longitudinal compressive strain with (C).

The experimental results showed an almost linear load-strain relationship and a good agreement with the predicted load-strain relationship based on FMA and FE. The results verified that the strains in both tension and compression increased linearly with load at the early stage of load application. The result also verified that longitudinal strain at the top of the sandwich beam matches the strain at the bottom. This showed that the assumption of compatibility of strains throughout the depth of the sandwich section, and the equilibrium of internal force resultants are valid. In general, the results showed that a simplified FMA can reasonably predict the load-strain behaviour of sandwich beams. The difference in the FMA, FE and experimental results is less than 5%. The small discrepancy observed could be attributed to the variations in the dimensions of the glue-laminated composite sandwich beam specimens.
6. CONCLUSIONS

In this paper, a simplified FMA is presented to describe the flexural behaviour of fibre composite sandwich beams with a high strength core material. Based on the results of the theoretical evaluation, the following conclusion can be drawn:

(1) Theoretical prediction and numerical simulations of the flexural behaviour of the composite sandwich beams considering the appropriate constitutive material behaviour were in good agreement with the experiment.

(2) The contribution of the high strength core material in the flexural and shear stiffness should be included to determine the flexural behaviour of the composite sandwich beam.

(3) Consideration of the effect of core cracking in tension in the calculation of the flexural stiffness provided a better understanding of the overall behaviour of sandwich beams.

(4) The FMA agreed very well with the experimental results and finite element analysis, showing the efficiency and practical application in analysing sandwich structures with phenolic core under flexural load.

(5) The suitability of the FMA for sandwich structures with other material systems and geometries are now being investigated for the adoption of this methodology for the general design of sandwich structures.

REFERENCES


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