EVALUATION OF EXISTING MODEL FOR PREDICTING OF FLEXURAL BEHAVIOUR OF GFRP-REINFORCED CONCRETE MEMBERS

I. M. Metwally
Reinforced Concrete Dept., Housing & Building Research Centre,
P.O.Box 1770 Cairo, Egypt, E-Mail: dr_ibrahimmetwally@yahoo.com, Mobile: 002-0102683991

ABSTRACT

Ashour formula [1] for predicting the flexural capacity of glass fiber-reinforced polymer (GFRP) bars reinforced concrete beams is introduced. An implementation of the Ashour model is presented for the purpose of verification and evaluation against the experimental results which were collected from the literature. This paper reported test results of seventy one concrete beams reinforced with GFRP bars subjected to a four point loading system. All beams were governed by two modes of flexural failure; GFRP bars rupture and concrete crushing. The present study introduces an important modification in Ashour formula to compute the flexural capacity of GFRP bars reinforced concrete beams with high accuracy. The comparisons between the theoretical flexural capacities which were computed by the modified equation and that experimentally measured show a very good agreement. The new proposed equation is not limited to GFRP bars only, but can be used for other FRP bars such as carbon fiber-reinforced polymer and aramid fiber-reinforced polymer. Influences of GFRP reinforcement ratio and the failure type on the beam deformability are also presented.

KEYWORDS
Ashour model; GFRP; flexural capacity; deformability factor

INTRODUCTION

Deterioration of reinforced concrete structures has become a serious problem in the last decade. This situation is mainly due to corrosion of steel reinforcing bars embedded in concrete. The inherent incompatibility that exists between concrete and steel reinforcement, which mainly arises from the corrosion of steel, led to the development of new concrete reinforcing materials. With their high strength and corrosion resistance, fiber-reinforced polymer (FRP) bars represent an alternative to steel reinforcement. Low modulus of elasticity, low ductility, and high cost are the main reasons why FRP bars in concrete structures have received limited attention.

The analytical procedure developed for the design of concrete structures reinforced with steel bars is not necessarily applicable to structures reinforced with FRP. The geometrical shape, ductility, modulus of elasticity, and bond characteristics of FRP bars are likely to be different from those of steel bars. Thus, the behavior of FRP reinforced concrete should be independently investigated.

The design of concrete sections in flexure that are reinforced with fiber reinforced polymers (FRP) is different from that of sections reinforced with steel because of the difference in mechanical properties of FRP and steel. Generally, the FRP bars used as reinforcement in concrete have tensile strength varying between 500 and 2200 MPa and modulus of elasticity varying between 40 and 150 GPa. The stress–strain relationship for FRP is linear up to rupture when the ultimate strength is reached. Unlike steel reinforcing bars, FRP bars do not undergo yield deformation or strain hardening before rupture. For this reason, design of sections in flexure has been based upon consideration of ultimate strength, serviceability, and deformability. The purpose of design for deformability is to ensure that failure of a section in flexure occurs only after development of sufficiently large curvature.

Several experimental studies [2-18] have been performed to understand the behavior of concrete beams reinforced with FRP bars. As a result, few design guidelines [19, 20, 21] have been published to aid the design and construction of concrete structures reinforced with FRP bars. However, these design guidelines emphasized the need for more research to validate the performance of FRP-reinforced concrete beams in flexure and shear.

The principal aims of this paper may be summarized as follows:
1. To present, show, and discuss the test results of 71 concrete beams reinforced with GFRP bars;
2. To validate the accuracy of the proposed equation by Ashour [1] for flexural capacity prediction;
3. To modify the Ashour formula to the new proposed equation which can be used to compute the moment capacity of FRP bars reinforced concrete beams with high accuracy;
4. To study and establish the member deformability with respect to GFRP reinforcement ratio.

TEST SPECIMENS

The test specimens consisted of 71 GFRP reinforced concrete beams collected from the literature. The parameters of this study were beam geometry (width & depth, b & d), amount of GFRP reinforcement (\(\rho_f\), %), and concrete compressive strength (\(f_{cu}\)). All beams were tested under two equal concentrated loads. All beams were designed to fail under flexure. The flexural failure is mainly occurred due to concrete crushing or tensile rupture of GFRP rebars either within the mid-span region (under pure bending) or under the applied point loads.

THEORETICAL PREDICTION OF FLEXURAL CAPACITY OF GFRP-REINFORCED CONCRETE BEAMS BY ASHOUR MODEL

As FRP bars do not exhibit yielding, flexural failure of FRP-reinforced concrete beams is characterized by either concrete crushing (over-reinforced case) or FRP bar rupture (under-reinforced case). In the following, compatibility of strains and equilibrium of forces are employed for estimating the flexural capacity of FRP concrete sections.

The parabolic stress distribution and equivalent rectangular stress block of concrete in compression proposed by Egyptian Code [22] are adopted in the present analysis as shown in Fig. 1(c). The concrete compressive force \(C\) is then calculated from:

\[
C = k_1 k_2 f_{cu} b x
\]

As concrete is very weak in tension, concrete below the neutral axis is cracked and therefore ignored. Assuming that the stress-strain relationship of FRP bars is linear up to rupture, the force in the bottom FRP bars is estimated below:

\[
T_f = A_f f_f = A_f E_f \varepsilon_f
\]

The above equation is valid for different types of FRP bars, i.e., GFRP, CFRP, and AFRP.

The moment capacity \((M_f)\) of the section can be obtained by taking moments of forces about any horizontal axis of the section; for instance about the bottom FRP bars

\[
M_f = k_1 k_2 f_{cu} b x \left( d - \frac{k_2 x}{2} \right)
\]

Balanced Reinforcement Ratio
The flexural failure mode (either concrete crushing or FRP rupture) could be identified by comparing the actual FRP reinforcement used with the balanced reinforcement ratio. The balanced ratio of FRP reinforcement is determined when, at the instant of failure, both concrete crushes and FRP bars rupture simultaneously; that is, 
\[ \epsilon_c = \epsilon_{cu} = 0.003 \] (concrete crushing) and 
\[ \epsilon_f = \epsilon_{fu} \] (tensile FRP bar rupture), where \( \epsilon_{cu} \) and \( \epsilon_{fu} \) are the ultimate strains of concrete and FRP bars, respectively. In such case, the depth of the neutral axis, \( x_b \) is

\[ x_b = \frac{\epsilon_{cu} d}{\epsilon_{cu} + \epsilon_{fu}} = \frac{0.003 d}{0.003 + \epsilon_{fu}} \]  \[\text{------------------------ [4]}\]

and the balanced ratio \( \rho_b \) of FRP reinforcement is

\[ \rho_b = \frac{A_{fb}}{bd} = 0.6 \frac{\epsilon_{cu} x_b}{\epsilon_{fu} d} \]  \[\text{------------------------ [5]}\]

If the reinforcement ratio is below the balanced ratio \( (\rho_f < \rho_b) \), FRP bars rupture. Otherwise, \( (\rho_f > \rho_b) \), concrete crushing failure mode governs.

**Modification of the Ashour Model**

Fig. 2 shows the correlation between \( M_{exp} \) and \( M_f \) which calculated by Ashour formula (Eq.3). This relationship can be represented by power curve; its equation is 
\[ M_{exp} = 1.237 M_f^{0.944} \] , this equation gives a good representation with \( R^2 \) (correlation factor) = 0.982 and the average and standard deviation of the \( M_{exp} / M_f \) for the seventy one studied beams equal 1.027 and 13.7 % respectively as reported in Table1.

\[ M_{exp} = 1.237 M_f^{0.944} \]
\[ R^2 = 0.982 \]

![Figure 2. Correlation between experimental moment capacities for 71 studied beams and the predicted values by Ashour model](image)

To modify the above equation for the best simulation, the power of \( M_f \) has changed to 0.94239 instead of 0.944, hence the new modified formula (or the proposed formula) is 
\[ M_p = 1.237 M_f^{0.94239} \]; this equation gives the best correlation with \( R^2 \approx 1 \) with the average and standard deviation of \( M_{exp} / M_p \) equal 1.013 and 5.9 % respectively as shown in Table 1.

The new proposed modified equation can be re-written as follow:-

\[ M_p = 1.237 \left[ k_1 k_2 f_{cu} b x \left( d - \frac{k_2 x}{2} \right) \right]^{0.94239} \]  \[\text{------------------------ [6]}\]

This equation is valid to predict the moment capacity of RC members with rectangular section reinforced with FRP rebars in tension side.

Fig. 3 shows that \( M_{exp} / M_p \) (computed by the proposed equation) is very closed and near to the one line compared to \( M_{exp} / M_f \)(computed by Ashour model).
VERIFICATION OF THE PROPOSED EQUATION

To verify the proposed modification, Equation no.6 was compared to the test results of 71 specimens tested to date. These test data were collected from 13 different investigations. Table1 gives the data of 71 concrete beams reinforced with GFRP bars and failed in flexure; all were simply supported and were tested in four-point bending. The reinforcement ratio of the test specimens ranged between 0.12 and 3.6%. The concrete compressive strength ranged between 32.47 to 67.29 MPa, and the depths ranged between 152 to 550 mm.

Besides the predicted moment capacities according to the proposed equation (Eq. no. 6), the predicted moment capacities according to Ashour model (Eq. no. 3) are also presented in Table1. For 71 beams, the average of $M_{\text{exp}} / M_p$ for the proposed equation is 1.013 with a standard deviation equal 5.9 %. One the other hand, the corresponding values were 1.027 and 13.7 % respectively, for the Ashour method.

Fig. 4 shows that the level of accuracy of the moment capacity of GFRP- reinforced beams predicted by the proposed equation is consistent unlike the method of Ashour. The same observation can be made when the results of the proposed equation and those by Ashour were plotted versus the concrete strength, overall depth, and GFRP reinforcement ratio as shown in the following Patterns.

Figure4. Influence of different variables as $f_{cu}$, $d$, and $\rho_f \%$ on $M_{\text{exp}} / M \text{ calculated}$ by Ashour and proposed equations.
Across the range of variables included in the data set, the predictions of the Ashour model appear to have larger band width of the scattered results and higher level of conservatism compared to that of the proposed equation which appears near to the one line. Thus, proposed equation (no. 6) appears to be more accurate and reliable for predicting the moment capacity of concrete members longitudinally reinforced with FRP rebars.

Table 1. Comparison between the experimental and calculated flexural capacities

<table>
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<tr>
<th>No.</th>
<th>Beam notation</th>
<th>Width x overall depth, mm</th>
<th>$f_{cu}$, MPa</th>
<th>$\phi_f$, %</th>
<th>$\rho_b$, %</th>
<th>$M_{exp}$, kN.m</th>
<th>$M_{p}$, kN.m (Ashour Eq.1)</th>
<th>$M_{exp}/M_{p}$</th>
<th>$M_{exp}/M_{p}$ (Proposed Eq.4)</th>
<th>Experimental mode of failure</th>
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- **Ashour [1]**
- **Benmokrane et al. [6]**
- **Benmokrane et al. [5]**
- **Al-Sayed [3]**
- **Brown and Bartholomew [7]**

**Table Note**: This table compares the experimental and calculated flexural capacities for various beam notations, detailing the width, overall depth, concrete strength, reinforcement percentages, and experimental modes of failure. The values are expressed in various units such as MPa, %, kN.m, and mode of failure.
<table>
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<tr>
<th>No.</th>
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<th>Width x overall depth, mm</th>
<th>$f_{cm}$, MPa</th>
<th>$\rho_b$, %</th>
<th>$\rho_f$, %</th>
<th>$M_{exp}$/M_kN.m</th>
<th>$M_kN.m$ (Ashour Eq.1)</th>
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DUCTILITY

The ductility of a beam can be defined as its ability to sustain inelastic deformation without loss in its load carrying capacity prior to failure. Following this definition, ductility can be expressed in terms of deformation or energy absorption. In the case of steel reinforced beams, where there is clear plastic deformation of steel at yield, ductility can be calculated as the ratio of ultimate deformation to deformation at yield. With FRP reinforced beams, there is no yield point; consequently, this simple definition can not be applied. ACI Committee 440-2001 [19] reported that the ductility of the FRP reinforced beams can be evaluated by means of the deformability factor (DF), defined as the ratio of the energy absorption at ultimate (area under load-deflection curve up to ultimate load) to the energy absorption at service load (at the serviceability deflection limit of span / 180).

Table 2 shows the values of DF for 35 GFRP reinforced concrete beams covered the big range of $\rho_f$ from 0.14 % to 3.6%.

Table 2 and Fig. 5 also show that, DF for compression failures were observed to be in the range of 4 to 15, whereas for tension failure, the ratios where observed to be in the range of 2 to 7.

![Image](image-url)

Figure 5. Influence of GFRP reinforcement ratio and failure type on deformability factor

The graph in Fig. 5 indicates that the all sections reinforced with GFRP rebars and failed under compression (over reinforced sections) have DF exceeding a minimum value of 4 to ensure a ductile failure as specified by CSA-S6-00 [26]. Whereas, 55% of the sections failed due to GFRP rupture (tension failure), and the value of DF increases with the increase of $\rho_f$ till a certain value equal nearly 1.5% after this limit the DF decreases.

Table 2. Deformability factors of various GFRP- reinforced concrete beams

<table>
<thead>
<tr>
<th>No.</th>
<th>Beam notation</th>
<th>f_{cu}, MPa</th>
<th>$\rho_s$, %</th>
<th>DF</th>
<th>Experimental mode of failure</th>
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CONCLUSIONS

Based on this study, the following conclusions can be made:

1. In this study, the Ashour model has been modified to give a more accurate and simple mathematical model for predicting the moment capacity of RC members with rectangular section reinforced with FRP rebars. It has been developed and validated against the experimental results of 71 GFRP-reinforced concrete beams from different recent researches. The new proposed equation provided an excellent correlation with test results. It is not limited to GFRP bars only, but can be used for other types of FRP bars.

2. Compression failure is a better mode of failure than tension failure in GFRP-reinforced concrete beams. This observation is based on the following factors; compression failure attained higher moment capacity, relatively gradual failure, and higher deformability factor i.e. better member deformability.

3. This study indicates that the values of deformability factor increase with increase of GFRP-reinforcement ratio up to 1.5 %; after this limit the deformability factors decrease.

4. Deformability factor in the range of 4 to 15 were observed for the beams failing in compression, where a higher percent of tension reinforcement provided a higher deformability factors. Whereas, for tension failure, the ratios where observed to be in the range of 2 to 7.
5. From this study, 55% of GFRP-reinforced concrete beams failed under tension have deformability factors less than a minimum value of 4. Therefore, it is strongly recommended to check the deformability in design of FRP-under reinforced sections.

LIST OF SYMBOLS

<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$A_f$</td>
<td>area of bottom FRP bars</td>
</tr>
<tr>
<td>$A_{fb}$</td>
<td>area of FRP bars at balanced conditions</td>
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<tr>
<td>$b$</td>
<td>width of concrete beam</td>
</tr>
<tr>
<td>$C$</td>
<td>concrete compressive force</td>
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<td>$d$</td>
<td>effective depth of section</td>
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<td>$DF$</td>
<td>deformability factor</td>
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<td>$E_f$</td>
<td>elastic modulus of FRP bars</td>
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<td>$f_{cu}$</td>
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<td>$f_c$</td>
<td>cylinder compressive strength</td>
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<td>$f_{tu}$</td>
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<td>$h$</td>
<td>overall depth of test specimens</td>
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<td>$k_1$</td>
<td>ratio of the average compressive stress to the concrete cube strength</td>
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<td>$k_2$</td>
<td>ratio of the depth of the idealized rectangular stress block to the neutral axis depth</td>
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<td>$M_{exp}$</td>
<td>experimental moment capacity</td>
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<td>$M_f$</td>
<td>theoretical moment capacity of GFRP sections calculated by Ashour formula</td>
</tr>
<tr>
<td>$M_p$</td>
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<tr>
<td>$P$</td>
<td>total failure load</td>
</tr>
<tr>
<td>$P_{cr}$</td>
<td>total load at first visual crack</td>
</tr>
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<td>$T_f$</td>
<td>force in bottom FRP bars</td>
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<tr>
<td>$x$</td>
<td>depth of the neutral axis</td>
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<tr>
<td>$x_b$</td>
<td>depth of the neutral axis for balanced section</td>
</tr>
<tr>
<td>$e_c$</td>
<td>strain at the top compression level of the concrete section</td>
</tr>
<tr>
<td>$e_{cu}$</td>
<td>ultimate strain of concrete (=0.003)</td>
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<td>$\rho_b$</td>
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<td>$\rho_f$</td>
<td>FRP reinforcement ratio ($A_f/\pi b d$)</td>
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