FLEXURAL RC MEMBERS STRENGTHENED WITH MECHANICALLY FASTENED FRP LAMINATES: TEST RESULTS AND NUMERICAL MODELING

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ABSTRACT

This paper presents an experimental and numerical investigation aimed at advancing the knowledge of the behavior of reinforced concrete (RC) members strengthened in flexure with Mechanically Fastened (MF) Fiber Reinforced Polymer (FRP) laminates. The test matrix included four MF-FRP strengthened specimens with different combinations of laminate lengths and fastener layouts, a counterpart strengthened with externally bonded (EB) FRP laminate, and a control (unstrengthened) specimen. Test results presented herein emphasize the influence of the partial interaction between RC slabs and MF-FRP laminates on the flexural response arising predominantly from bearing of the fasteners onto the FRP laminate. A finite element procedure was developed to carry out the numerical study, which incorporates nonlinear constitutive models for materials and concrete-FRP interface. For the latter, an accurate and a simplified, conservative bilinear stress-slip model are successfully implemented and verified to evaluate applicability for analysis and design purposes, respectively.

KEYWORDS

FRP, fasteners, RC slabs, flexural strengthening, experimental tests, bearing slip, numerical modeling.

INTRODUCTION

The use of Mechanically Fastened (MF) Fiber Reinforced Polymer (FRP) laminates is emerging as a promising means for the flexural strengthening of reinforced concrete (RC) members (Lamanna 2002; Rizzo 2005). This technology relies on pre-cured FRP laminates with enhanced bearing strength that are connected to the concrete substrate by means of steel anchors, which makes it well suited for emergency repairs where constructability and speed of installation are critical requirements.

This paper presents an experimental and numerical investigation aimed at better understanding of the effect of alternative configurations of MF-FRP strengthening systems on the flexural response of RC slabs, as well as the influence of the relative displacement (“slip”) between the concrete substrate and the FRP laminate mainly accruing from bearing of the fasteners onto the laminate. This bearing-slip at concrete-FRP interface is generally neglected by existing simplified analytical procedures that predict the flexural response of MF-FRP strengthened RC members by assuming the conservation of plane sections (Lamanna 2002; Bank and Arora 2007).

The numerical procedure presented in this paper implements an iterative secant algorithm based on a general approach (Faella et al. 2002) previously adapted for the analysis of beams strengthened with externally bonded (EB) FRP laminates (Faella et al. 2008), which assumes nonlinear constitutive models for concrete, steel and concrete-FRP interface. For the latter, an accurate and a simplified, conservative bilinear stress-slip model are implemented and verified to evaluate applicability for analysis and design purposes.

EXPERIMENTAL INVESTIGATION

Experimental program

A total of six one-way RC slabs, 3658 mm (12 ft) long and with a cross section of 305 by 152 mm (12 by 6 in), were tested under four-point bending, as illustrated in Figure 1a. Four specimens were strengthened with a MF-FRP laminate, a counterpart was strengthened with an EB-FRP laminate, while an unstrengthened slab was used as control specimen. Pultruded composite strips “Saftrip®” manufactured by Strongwell (2009) were used for the MF-FRP strengthened slabs. Width and thickness of these laminates were 102 and 3.2 mm, respectively. Wedge-bolt screw anchors (Powers Fasteners 2008) made of zinc-plated carbon steel were used as fasteners (Figure 1b). The anchors had a 44.5 mm shank length and a 9.5 mm diameter, and were driven into pre-drilled holes using a...
common torque wrench. A unidirectional carbon fiber sheet with ply thickness of 0.165 mm and width of 305 mm was used for the specimens strengthened with the EB-FRP method, and applied via wet-layup with a two-part epoxy saturant (Basf Mbrace® 2008). The table in Figure 1c reports the main data of the strengthening systems, i.e.: width \((b_f)\), thickness \((t_f)\) and length \((L_f)\) of the FRP laminate; fastener layout (either Pattern No.1 or No.2 as shown in Figure 1d) and total number of bolts in the clear span \((N_b)\). In this table, the control specimen and the EB-FRP strengthened specimen are denoted as “C” and “EB”, respectively. The MF-FRP strengthened specimens are labeled as “MF-X-Y”, where \(X\) indicates the fastener layout (“1” for Pattern No. 1, “2” for Pattern No. 2), and \(Y\) indicates the laminate length (“L” for longer laminate, “S” for shorter laminate). It is worth noting that in Pattern No. 1, the number of fasteners is equal to the numbers of pre-drilled holes, while in Pattern No. 2, the fasteners are placed at every second hole (Figure 1d).

The large amount of data recorded during test by proper instrumentation of specimens allowed to investigate the effect of alternative configurations of MF-FRP strengthening systems on the flexural response of members in terms of strength, deflection and strain. In this paper, the authors emphasise the occurrence of significant “slips” at concrete-FRP interface mainly accruing from bearing of the fasteners onto the FRP laminate.

More details about test specimens, FRP strengthening systems and experimental results can be found in Napoli (2008) and Napoli et al. (2009).

### Test results: bearing-slip at concrete-FRP interface

The experimental readings of the compression strain in the concrete, and the tensile strain in the steel reinforcement and FRP laminate allowed to monitor the strain distribution throughout the midspan cross section of specimens at increasing values of the applied bending moment, and, consequently, to determine the respective position of the neutral axis which is computed under the assumption of conservation of plane sections. Figure 2 illustrates the position of the neutral axis for two representative specimens (MF-1-L and MF-2-L). In particular, in Figures 2(a), this position was determined by assuming a linear strain distribution throughout the cross section between the extreme concrete fiber in compression, \(\varepsilon_{c}\), and the extreme FRP fiber in tension, \(\varepsilon_{f}\). In Figures 2(b), instead, the position of the neutral axis was determined by assuming a linear strain distribution between, \(\varepsilon_{c}\), and the steel reinforcement, \(\varepsilon_{s}\), which must be satisfied in RC sections.

In the case of the Specimen MF-1-L, as the ultimate moment was attained \((M_u = 37.3 \text{ kN-m})\), Figure 2a shows a value of the neutral axis depth \(y_{NA} = 51 \text{ mm}\), measured from the extreme compression fiber, which is associated to the measured strains \(\varepsilon_{c} = 3310 \mu\varepsilon\) and \(\varepsilon_{f} = 6495 \mu\varepsilon\). It is noted that this value is significantly different from \(y_{NA} = 30 \text{ mm}\) shown in Figure 2b and computed by means of \(\varepsilon_{c} = 3310 \mu\varepsilon\) and \(\varepsilon_{s} = 10865\).

Similar observations can be drawn in the case of the Specimen MF-2-L for which the measured strains at ultimate moment \((M_u = 33.9 \text{ kN-m})\) were \(\varepsilon_{c} = 2180 \mu\varepsilon\), \(\varepsilon_{f} = 4055 \mu\varepsilon\) and \(\varepsilon_{s} = 10331 \mu\varepsilon\).

The mismatch stems from significant differential deformations (“slip”) at the concrete-FRP interface, which initiated at the early loading stages and significantly increased as testing proceeded. Different components may contribute to the slip mechanism, including: gaps between fasteners and surrounding concrete and, to a larger extent, between fasteners and FRP laminate; elastic deformation of the fasteners under shear forces; and bearing mechanism at the hole locations in the FRP laminate. These phenomena initiate at different loading stages, and
their relative influence changes throughout the loading process: for example, bearing of the fasteners onto the FRP laminate necessarily initiates after the gaps between fasteners and surrounding FRP are filled, and reasonably becomes predominant past yielding of the steel reinforcement.

The experimental evidence gained shows that the bearing-slip occurring at FRP-concrete interface cannot be neglected in the analysis and design of MF-FRP strengthened members at both service and ultimate load conditions, whereby the assumption of conservation of plane sections may lead to inaccurate predictions. In fact, based on this assumption, the maximum longitudinal strain in the FRP in Specimen MF-1-L should be comparable to that in the steel reinforcement, whereas the measured values indicate that the former is about 40% smaller than the latter. These predictions may be even more inaccurate when Pattern No. 2 is used for which, as experimentally observed, the occurrence of the bearing-slip at concrete-FRP interface leads to a maximum longitudinal strain in the FRP significantly smaller than that in the steel rebars.

**Numerical Investigation**

The numerical model used for simulating the behavior of RC beams externally strengthened with MF-FRP laminates is based on a general algorithm formulated by implementing the differential equation of the well-known Newmark’s theory for steel-concrete composite beams with linear-elastic shear connectors (Newmark et al. 1951) into the “exact” finite element developed for the analysis of partial interaction in composite beams with flexible shear connectors (Faella et al. 2002). The fundamentals of this approach and the basic assumptions made to implement the numerical procedure are reported in the following subsections.

**Newmark’s model: highlights of the formulation**

The following assumptions are introduced to model RC beam, MF-FRP laminate, and concrete-FRP interface in the linear-elastic range: 1) the flexural stiffness of the FRP laminate is neglected, and only its axial stiffness is considered; 2) the interaction between RC beam and MF-FRP laminate is modeled as a continuous, linear medium of negligible thickness; 3) equal vertical displacements occur in the connected RC slab and FRP laminate elements; 4) shear deformations of the RC slab are neglected.

Relative displacements (slips) $s$ are produced at the concrete-FRP interface due to the partial interaction between RC slab and FRP laminate, as illustrated in Figure 3a, where

$$s = u_{c,i} - u_{f,j} = \left( u_c + \phi y_{c,i} \right) - \left( u_f - \phi \frac{t_f}{2} \right) = u_c - u_f + \phi d_f$$

(1)
The axial forces acting on the RC slab and the FRP laminate, applied at the center of gravity of slab and laminate section, indicated with $G_c$ and $G_f$, respectively, are denoted with $F_c$ and $F_f$, respectively. The applied bending moment $M$ at a given cross section can be expressed as:

$$M = M_c + F_f \cdot d_f = \chi \cdot E I_c + F_f \cdot d_f$$

(2)

where $M_c$ is the bending moment in the RC slab, $\chi$ is the associated section curvature, and $EI_c$ is the flexural stiffness of the cross section of the slab. The longitudinal shear force per unit length at the concrete-FRP interface is rendered as a function of the slip $s$ in the following form:

$$\frac{dF_f}{ds} = k \cdot s$$

(3)

where $k$ is the stiffness parameter of the concrete-FRP interface. Based on the compatibility relation given in Eq. (1), the equilibrium relation given in Eq. (2), and the interface relation given in Eq. (3), the following second-order differential equation is derived in terms of curvature:

$$\frac{d^2\chi}{dz^2} - \alpha^2 \chi = -\frac{q}{EI_c} - \alpha^2 \frac{M}{EI_{tot}}, \quad \alpha^2 = \frac{k \cdot EI_{tot}}{E A^* \cdot EI_c}$$

(4)

where $EI_{tot}$ denotes the flexural stiffness of the MF-FRP strengthened RC cross section under the hypothesis of full composite action, which is defined as

$$EI_{tot} = EI_c + \left(\frac{E f A f}{E f A f + E A}\right) d_f^2 = EI_c + E A^* d_f^2$$

(5)

where $E A_c$ and $E A_f$ are the axial stiffness of the RC slab and FRP laminate cross section, respectively; $q$ is the arbitrary transverse load distributed along the longitudinal axis of the beam; and $E A^*$ denotes the effective axial stiffness of the MF-FRP strengthened RC section.

**Finite element formulation of MF-FRP strengthened RC beam element**

A two-node finite element can be formulated (Faella et al. 2002) by implementing the “exact” solution in Eq (4) for the analysis of MF-FRP strengthened RC beams. In particular, the typical relation of flexibility-based finite elements can be derived for simply supported MF-FRP strengthened beam elements in the form

$$\delta = DX + \delta_0$$

(6)

where $X = [M_c, F_f, M_f, F_f]$ and $\delta = [\phi_i, s_i, \phi_j, s_j]$ are the vectors of nodal forces and nodal displacements, respectively, as illustrated in Figure 3b (left side). The displacement-based relation that relates nodal forces and nodal displacements for an unrestrained MF-FRP strengthened beam, as shown in Figure 3b, (right side), can also be obtained by inverting the flexibility matrix $D$. The relation between the vector of nodal forces, $Q = [V_i, M_c, F_f, V_f, M_f, F_f]$], and the vector of nodal displacements, $s = [\phi_i, s_i, \phi_j, s_j]$, is given as

$$Q = Ks + Q_0$$

(7)

where $K$ is the stiffness matrix, and the vector $Q_0 = [V_{oi,q}, M_{tot,q}, F_{oi,q}, V_{oj,q}, M_{tot,q}, F_{oj,q}]$ collects the nodal forces produced by the arbitrarily distributed load $q$.

**Nonlinear analysis procedure**

The two-node finite element introduced can be used for nonlinear analysis through fiber discretization of the beam cross section, and by implementing an iterative convergence procedure based on the “secant” value approach to account for material nonlinearity, including concrete, steel, and concrete-FRP interface, as demonstrated earlier for the case of EB-FRP strengthened RC members (Faella et al. 2008). The original constitutive model for concrete (Faella et al. 2008) is assumed, whereas a modified bilinear stress-strain constitutive model for the steel rebars is adopted herein where strain-hardening is rendered by means of a constant modulus ratio of 0.03 between the elastic modulus in the plastic and the elastic range, respectively.

Figure 3. Schematic of internal forces and deformations in cross section of MF-FRP strengthened RC slab (a); nodal forces and displacements in two-node beam finite element (b)
Since the assumption of conservation of plane sections was shown to be inaccurate, a suitable stress-slip model for the concrete-FRP interface must be defined and implemented in the numerical procedure. First, as the discrete connections are made by fasteners spaced at a longitudinal distance \( i \) on-center, the following approximation is introduced in Eq. (3) for the longitudinal shear force per unit length:

\[
\frac{dF_f}{dz} \approx \frac{P(s)}{i}
\]  

(8)

where \( P(s) \) is the shear force transferred through a single fastener for the local value of slip, \( s \), approximated as:

\[
P(s) = k(s) \cdot s \equiv \sigma_f(s) \cdot t_f \cdot d_i
\]  

(9)

where \( k(s) \) is the nonlinear stiffness parameter of the concrete-FRP interface, \( \sigma_f(s) \) is the bearing stress produced by the fastener in the FRP laminate and \( d_i \) is the diameter of the fastener. The recently proposed bearing stress-slip (\( \sigma_f(s) \)-s) model in Eq. 10 is implemented, which is defined based on regression analysis performed on the results of shear tests on concrete blocks with FRP laminates connected by means of screw anchors (Elsayed et al. 2009). In Eq. 10, \( f_{bu} = 234 \) MPa is the stress in the FRP laminate as bearing initiates, resulting in a deviation from linearity of the stress-slip relation, and \( s_b = 2 \) mm is the associated slip; \( f_{bu} = 385 \) MPa is the bearing strength of the FRP laminate, and \( s_{bu} = 9 \) mm is the associated slip; and \( A = s/s_b \). The bilinear \( \sigma_f(s) \)-s model given in Eq. 11 was also evaluated for use in design, where the more accurate nonlinear model was simplified in a conservative fashion by assuming a linear response up to \( f_{bu} = s_{bu} \). Eqs. 10 and 11 are schematically plotted in Figure 4.

The proposed numerical procedure is pursued until one of the following failure modes is attained: a) concrete crushing, which occurs as the maximum strain in the concrete, \( \varepsilon_c \), reaches its ultimate value, \( \varepsilon_{cu} = 0.004 \); b) yielding followed by rupture of the steel reinforcement, which occurs as the maximum steel tensile strain, \( \varepsilon_s \), reaches the rupture limit; c) FRP tensile failure, which occurs as the maximum interface slip reaches a value of 20 mm in agreement with experimental measurements reported in Elsayed et al. (2009).

Figure 4. Modeling of bearing stress-slip: accurate model for analysis (a), and simplified model for design (b)

**NUMERICAL RESULTS AND DISCUSSION**

Representative comparisons between numerical and experimental midspan displacement curves with respect to the applied maximum bending moment for MF-FRP strengthened specimens are shown in Figure 5, including the results from both the bearing stress-slip models in Eqs. 10 and 11. Irrespective of the bearing-slip model implemented, crushing of the concrete was the predicted failure mode for all specimens, in agreement with the experimental results (Napoli et al. 2009). When the interface model in Figure 4a (Elsayed et al. 2009) is implemented, the proposed numerical procedure yields moment-displacement curves in very good agreement with those measured experimentally for the case of Pattern No. 1. As the fastener spacing is increased as in Pattern No. 2, the post-cracking stiffness up to yielding of the steel reinforcement is approximated less accurately compared to Pattern No. 1. A reasonable explanation may be found in the fact that such configuration results in greater shear forces carried by each fastener, which may produce a more extensive local cracking at the hole locations at relatively low loads, with consequent marginal reduction in the global flexural stiffness. In fact, a slightly smaller post-cracking stiffness could be measured when using Pattern No. 2, as it is seen by comparing the experimental curves of Specimens MF-1-L and MF-2-L in Figure 5. Still, very good results are obtained in terms of strength and displacement at failure. In all cases, the use of the interface model in Eq. (11) leads to more conservative strength results, consistent with its purpose. Therefore, this model may be implemented in the proposed numerical procedure to evaluate design alternatives.
CONCLUSIONS

In this paper, the flexural behavior of RC members strengthened in flexure with MF-FRP laminates has been investigated. Six RC one-way slabs were tested to failure in four-point bending; the test matrix included a benchmark unstrengthened slab, a slab strengthened by means of an EB-FRP laminate, and four MF-FRP strengthened specimens with different laminate length and fastener patterns. From test results it has been shown that the bearing-slip that develops at FRP-concrete interface should not be neglected in the analysis and design of MF-FRP strengthened members, whereby the assumption of conservation of plane sections may lead to inaccurate predictions. A numerical procedure for simulating the flexural behavior of the MF-FRP strengthened slabs has been also presented, which incorporates nonlinear constitutive models for concrete and steel materials, and concrete-FRP interface. For the latter, an accurate and a simplified, conservative bilinear stress-slip model have been implemented. The proposed procedure yields predictions that are in good agreement with the experimental results when using the accurate nonlinear interface bearing-slip model. Conservative results are obtained when using the simplified (bilinear) model, making it suitable for design purposes.

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