ELASTIC MOMENT CAPACITY AND PRESTRESS LOSS OF STEEL-CONCRETE COMPOSITE BEAMS STRENGTHENED WITH A PRESTRESSED CFRP PLATE

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ABSTRACT

Many studies have confirmed that externally-bonded carbon fibre reinforced polymer (CFRP) plate can effectively improve the stiffness and strength of steel-concrete composite beams. It has been suggested that prestressed strengthening is the more economical alternative to unstressed strengthening. Experiments and applications have shown the flexural stiffness and moment capacity of beams were improved significantly by the use of the prestressing methods. In this paper, a solution is presented to calculate the elastic moment capacity of composite beams Strengthened with a prestressed CFRP plate. Two prestressing methods, prestressed FRP and load-relief jacking, are considered. The solutions of effective stress and prestress loss for the both of prestressing methods are presented as well. It was found that the prestress loss ratios of the two methods are same. A parametric study was carried out to show how the elastic moment capacity and prestress loss are influenced by the dimensions of the section and the material properties of the strengthened beam.

KEYWORDS

CFRP plate, prestress, steel-concrete composite beam, strengthening, elastic moment capacity

INTRODUCTION

In recent years, Carbon fibre reinforced polymer (CFRP) materials have made tremendous advanced in building and bridge upgrade and rehabilitation. The most popular development is the use of CFRP in the repair and upgrading beams. Recent research (e.g. Hollaway and Leeming 1999; Deng et al. 2004; Deng and Lee 2007; Sen et al. 2001) has now established the effectiveness of strengthening concrete beams, steel beams and steel-concrete composite beams by bonding CFRP plate. If the CFRP plate is simply bonded to the tensile face of the beams, however, the strengthening can only carry live loads applied to the beams after it has been strengthened and cannot carry the dead loads (El-Hacha et al. 2004). To improve the effectiveness of this strengthening technique and the CFRP materials, two prestressing schemes can be used to transfer a portion of the permanent load from the beam into the CFRP plate (Cadei et al. 2004). They are prestressed FRP method and load-relief jacking method, as shown in Figure 1. Experimental study (Quantril and Hollaway 1998) confirmed prestressing the plate caused a considerable increase in the ultimate moment capacity and stiffness of the concrete beams in comparison to the nonprestressed CFRP plate. Guo et al. (2006) conducted a theoretical study on the ultimate bearing capacity and prestress loss of the concrete beams Strengthened with prestressed FRP sheets. Smith (2004) confirmed the bearing capacity of the metallic beams effectively improved by using load-relief jacking scheme. In this paper, an analytical study of elastic moment capacity of the steel-concrete composite beams under mechanical and thermal loads is reported. The effect of the dead load applied on the beam prior to the bonding and the prestress schemes is considered as well. Moreover, a solution for effective prestress and the prestress loss is presented for the both of prestress schemes. A parametric study for bearing capacity and prestress loss is carried out.

Figure 1. Schematic two prestressing schemes

(a) Prestressed FRP

(b) Load-relief jacking

Ff

beam

FRP

beam

FRP
METHOD OF SOLUTION

Longitudinal Tension and Bending Moment of the Beam

All materials are considered linear elastic. The control cross section is not close to the end of the CFRP plate. Deng et al. (2004) indicated that the interfacial shear stress can be ignored if the cross section is around 100 mm far from the end of the plate. Therefore, the strain at the bottom of the beam \( \varepsilon_b \) is equal to the strain at the top of the CFRP plate \( \varepsilon_p \). \( \varepsilon_b \) and \( \varepsilon_p \) can be expressed as:

\[
\varepsilon_b = \frac{M_b Z_b + N_b}{E_b A_b} + \alpha_b \Delta T - \frac{M_f Z_b}{E_b I_b} \\
\varepsilon_p = -\frac{M_p Z_p + N_p}{E_p A_p} + \alpha_p \Delta T - \frac{F_f}{E_p A_p}
\]

where \( M \) and \( N \) are bending moment and longitudinal tension, respectively, \( Z_b \) and \( Z_p \) the distance from the neutral axis of the beam to the bottom of the beam and the distance from the neutral axis of the CFRP plate to the upper surface of the CFRP plate, respectively, \( \alpha \) and \( \Delta T \) the thermal expansion coefficient and the temperature change, respectively, \( E, I \) and \( A \) the elastic modulus, the second moment of area and the area, respectively, \( M_f \) the bending moment caused by load-relief jacking, \( F_f \) the prestressing force. The subscripts \( b \) and \( p \) denote beam and CFRP plate.

The force equilibrium of the cross section gives

\[
N_p = -N_b
\]

Ignoring the bending moment in the plate \( M_p \), the moment equilibrium of the cross section gives

\[
M_b = M_1 - N_p (Z_b + Z_p)
\]

where \( M_1 \) is the bending moment only caused by the live load. Therefore, the bending moment caused by the dead load applied on the beam prior to the strengthening \( M_b \) is not counted in \( M_b \). Substituting (3) and (4) into \( \varepsilon_b = \varepsilon_p \) gives:

\[
N_p = \frac{1}{s} \left( \frac{Z_b}{E_b I_b} M_1 + (\alpha_b - \alpha_p) \Delta T + \frac{F_f}{E_p A_p} - \frac{M_f Z_b}{E_b I_b} \right)
\]

\[
M_b = \frac{1}{s} \left( (\frac{1}{E_b A_b} + \frac{1}{E_p A_p}) M_1 - (Z_b + Z_p)((\alpha_b - \alpha_p) \Delta T + \frac{F_f}{E_p A_p} - \frac{M_f Z_b}{E_b I_b}) \right)
\]

where

\[
s = \frac{(Z_b + Z_p)Z_b}{E_b I_b} + \frac{1}{E_b A_b} + \frac{1}{E_p A_p}
\]

In Eqs. (5) and (6), \( M_1 \) is negative and \( F_f \) is positive. Therefore, the longitudinal tension \( N_p \) increases with temperature, load-relief jacking force and prestressing force, but the bending moment \( M_b \) decreases with temperature, load-relief jacking force and prestressing force. Moreover, the strain at the bottom of the beam caused by the thermal change \((\alpha_b - \alpha_p) \Delta T \) and load-relief jacking \((-M_f Z_b/(E_b I_b)) \) can be equalized to the strain of the CFRP plate caused by the prestressing force \( F_f/E_p A_p \).

Elastic Moment Capacity of the Strengthened Beam

Assuming the cross section remaining plane, the section analysis of the strengthened beam is shown in Figure 2. In the figure, \( f_u \) is the yielding stress (ultimate elastic strength) of the steel beam, \( \sigma_p \) is the tensile stress of the CFRP plate. The tensile yielding on the bottom of the steel beam is the main failure mode of the steel-concrete composite beams if non-elastic strain is not allowed in the beam. The ultimate tensile stress of the bottom of the beam is \( M_u Z_b/(E_b I_b) \), where \( M_u \) is the elastic moment capacity of non-strengthened beams. Since the ultimate tensile strength of CFRP is much larger than that of steel, the tensile yielding on the bottom of the beam is also the main failure mode of strengthened beams. The failure critical equation of the strengthened beams can be written as:

\[
\frac{M_u Z_b}{E_b I_b} + \frac{M_u Z_b}{E_u I_b} + \frac{N_b}{E_u A_u} = \frac{M_u Z_b}{E_u I_u}
\]

Substituting Eqs. (5) and (6) into Eq. (8) gives:
In Eq. (9), $Z_p$ and $\alpha_p$ are ignored. The first item on the right of Eq. (9) shows the effect of the dimensions of the cross section on the elastic moment capacity. The second item on the right of Eq. (9) shows the effect of the thermal change, prestressing force and load-relief jacking. The elastic moment capacity of the strengthened beam $M_{ru}$ is:

$$M_{ru} = M_1 + M_0 \tag{10}$$

When the bending moment is equal to the elastic moment capacity $M_{ru}$, the section area of the CFRP plate required is:

$$A_p = \frac{E_p}{\alpha_p} \left( \frac{M_{ru} - M_u - (Z_b + I_b/Z_b A_b) F_f}{M_u - M_u - M_f + \alpha_b \Delta T Z_b/A_b} \right) \left( \frac{1}{Z_b A_b/1 + 1} \right) A_b \tag{11}$$

**Effective Prestress and Prestress Loss**

Only the prestress loss caused by the bending and compression of the beam is considered in this study.

(1) Prestressed FRP scheme

The prestress of CFRP plate can be expressed as $\sigma_{p0} = F_f/A_p$. When only prestressing force $F_f$ is considered in Eq. (5), the effective prestress $\sigma_{pc}$, which is equal to the longitudinal tension of CFRP $N_p$ divided by the area of the cross section of CFRP $A_p$, can be expressed as:

$$\sigma_{pc} = \frac{\sigma_{p0}}{s E_p A_p} \tag{12}$$

So the prestress loss $\sigma_{p1}$ and the loss ratio $c_p$ can be given as:

$$\sigma_{p1} = \sigma_{p0} - \sigma_{pc} = \left(1 - \frac{1}{s E_p A_p}\right) \sigma_{p0} \tag{13}$$

$$c_p = \frac{\sigma_{p1}}{\sigma_{p0}} = 1 - \frac{1}{s E_p A_p} \tag{14}$$

(2) Load-relief jacking scheme

The stress on the bottom of the beam caused by the jacking can be given as $\varepsilon_{bo} = M_j / Z_b E_p I_b$. If this stress is equalized to the stress caused by prestressed FRP, the equalized prestress $\sigma_{bo}$ is given by:

$$\sigma_{bo} = \frac{M_j Z_b}{E_p I_b} \tag{15}$$

If only bending moment caused by the jacking $M_j$ is considered in Eq. (5), the effective prestress $\sigma_{bc}$, which is equal to $N_p$ over $A_p$, is given by:

$$\sigma_{bc} = \frac{1}{s E_p A_p} \sigma_{bo} \tag{16}$$

So the prestress loss $\sigma_{b1}$ and the loss ratio $c_b$ can be given as:
\[ \sigma_{b1} = \sigma_{b0} - \sigma_{bc} = (1 - \frac{1}{sE_pA_p})\sigma_{b0} \quad (17) \]
\[ c_b = \frac{\sigma_{b1}}{\sigma_{b0}} = 1 - \frac{1}{sE_pA_p} \quad (18) \]

From Eqs. (14) and (18), it can be noted that the loss ratio \( c \) of these two methods are the same. This is because the stress caused by the load-relief jacking is equalized to that caused by the prestressed FRP. If the equalized prestress \( \sigma_{b0} \) is equal to the prestress in CFRP plate \( \sigma_{p0} \), the effective prestress and prestress loss of these two methods are the same.

EXAMPLE AND DISCUSSIONS

An example is employed to discuss the results and carry out a parametric study. The steel-concrete composite beam is strengthened with a 4 mm thick CFRP plate and the parameters are shown in Table 1. The elastic moment capacity of the composite beam \( M_e \) is equal to 147.36kNm.

<table>
<thead>
<tr>
<th>Table 1 Section parameters and mechanical properties of materials</th>
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<tbody>
<tr>
<td>Moment of inertia ( I ) (mm(^4))</td>
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<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>Composite beam ((b))</td>
</tr>
<tr>
<td>CFRP plate ((p))</td>
</tr>
</tbody>
</table>

Note: when the moment of inertia and area of the composite beam is calculated, the cross section of concrete slab is equivalent to the section of steel beam.

For prestressed FRP scheme, the CFRP plate is prestressed to 20% of tensile strength of CFRP; for load-relief jacking scheme, the moment caused by the jacking is applied to 20% of the elastic moment capacity of the composite beam. Ignoring the dead load and thermal change, the elastic moment capacities of the strengthened beam for two prestressing schemes are calculated using Eq. (9), respectively, as shown in Table 2. The results show the elastic moment capacity are improved by prestressed strengthening, but the improvement by the load-relief jacking is only 36.7% of that of the prestressed FRP. Therefore, the strengthening efficiency of the latter is better than the former.

A parametric study of Eq. (9) shows that elastic moment capacity increases linearly with the area and Young’s modulus of CFRP, temperature and prestress. Therefore, it is beneficial for the elastic moment capacity to increase the area and Young’s modulus of CFRP, temperature and prestress. Since \( f_t \) in Eq. (9) is constant along the CFRP plate, the improvement of the moment capacity by prestressed FRP is same in the whole strengthened area with the 100 mm length area from the plate end. But \( M_f \) in Eq. (9) vary along the CFRP plate. The moment capacity is improved mostly in the position applied jacking, where the moment caused by the jacking is the maximum.

<table>
<thead>
<tr>
<th>Table 2. Elastic moment capacity of beams</th>
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<tbody>
<tr>
<td>Non-strengthened</td>
</tr>
<tr>
<td>-----------------</td>
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<tr>
<td>Elastic moment capacity (kNm)</td>
</tr>
<tr>
<td>Improvement (%)</td>
</tr>
</tbody>
</table>

The effective prestress, prestress loss and loss ratio for two prestressing schemes are calculated by Eqs. (12-18), as shown in Table 3. The loss ratios of two schemes are same. It can be found in Eqs. (14) and (18) that the loss ratio is only influenced by the dimensions of the cross-section and the material properties, not by the loading and prestress. The parametric study for Eq. (14) and (18) shows that the loss ratio increases with the Young’s modulus and area of the CFRP plate, but decreases with the stiffness and area of the beam.

<table>
<thead>
<tr>
<th>Table 3. Analyses of prestress</th>
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<tbody>
<tr>
<td>Prestress ( \sigma_0 ) (N/mm(^2))</td>
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<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Prestressed FRP method</td>
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<tr>
<td>Load-relief jacking</td>
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</tbody>
</table>
CONCLUSIONS

In this paper, a solution is presented to calculate the elastic moment capacity and the required section area of CFRP plate of steel-concrete composite beam strengthened with a prestressed CFRP plate. Two prestressing methods, prestressed FRP and load-relief jacking, are considered. The solutions for effective stress and prestress loss for the both of prestressing methods are presented as well. The results and parametric study shows: 1) the elastic moment capacity can be improved by CFRP strengthening and it increased with the section area and Young’s modulus of the CFRP plate; 2) the both of prestressing schemes improved the elastic moment capacity further and the effectiveness of the CFRP material; 3) the prestress loss is only influenced by the dimensions of the section and the material properties, not by the prestress; 4) the prestress loss of two prestressing schemes are same, but the strengthening effectiveness of the prestressed FRP scheme is better than that of the load-relief jacking scheme.

The equations and conclusion obtained in this study can be used for steel or concrete beams strengthened with prestressed FRP as well.

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