MECHANICAL PERFORMANCE OF CURVED FRP REBARS - PART II:
ANALYTICAL STUDY

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ABSTRACT

Findings from an experimental study conducted by the authors (Mechanical performance of curved FRP rebars -
part I: Experimental study) confirmed that the tensile strength of curved FRP bars can be largely reduced under a
combination of a multiaxial state of stress. This phenomenon can often become an issue when curved
unidirectional composite elements are used to reinforce concrete structures. This is especially critical when the
fibres are designed to carry high tensile stresses, since premature failure can occur at the corner portion of the
composite. Tests carried out by the authors have shown that the tensile strength of the bent portion of composite
bars can be as low as 25% of the maximum tensile strength that can be developed in the straight part. In a
significant number of cases, the current design recommendations for concrete structures reinforced with FRP
were found to overestimate the bend capacity of tested FRP rebars. This paper presents and discusses potential
constraints relating to the use of curved FRP bars in concrete and proposes a macromechanical failure based
model to predict the ultimate capacity of a bent portion. The proposed model is then validated against test data
obtained from pullout tests on FRP bent bars.

KEYWORDS

Curved FRP, Bend capacity, Failure criteria, Macromechanics, Design.

INTRODUCTION

Steel reinforcement in concrete has the tendency to corrode and this process can lead to structural damage. FRP
reinforcement represents a viable alternative for structures exposed to aggressive environments and has many
possible applications where superior corrosion resistance properties are required. The use of FRP rebars as
internal reinforcements for concrete, however, is limited to specific structural elements and does not yet extend
to the whole structure. The reasons for this relate to the limited availability of curved or shaped reinforcing
elements on the market and their reduced structural performance. Various studies, in fact, have shown that the
mechanical performance of bent portions of composite bars is reduced significantly under a multiaxial
combination of stresses and that the tensile strength can be as low as 40% of the maximum tensile strength that
can be developed in the straight part (Guadagnini et al. 2006; Imjai et al. 2006; Ishihara et al. 1997; Morphy et al.
1997). The reduction in strength that occurs at the corners of a FRP bar has been quantified using empirical
models such as that proposed by the Japanese Concrete Institute (1997), which is described by Eq (1). In this
equation, the strength of the bent portion, $\sigma_b$, is expressed solely as a function of the uniaxial tensile strength of
the composite, $\sigma_{1,\text{max}}$, and the bar geometry (i.e. bar diameter, $d$, and bend radius, $r$).

$$\sigma_b = \left(\alpha \frac{r}{d} + 0.3\right) \sigma_{1,\text{max}} \leq \sigma_{1,\text{max}}$$

(1)

The value of $\alpha=0.05$ corresponds to a 95% confidence limit, whilst $\alpha=0.092$ corresponds to a 50% confidence
limit. Eq (1) yields generally a conservative estimate of the maximum strength that can be developed in bent bars
and it is currently adopted in the different design recommendations for FRP RC structures proposed by the
American Concrete Institute Committee 440 (2006); ISIS Canada (2001) and the Institution of Structural
Engineers (1999).

A MACROMECHANICAL BASED FAILURE MODEL

It is proposed that an analytical model based on macromechanical principles could adequately capture the true
degradation of the strength of bent composites. When a reinforcing bent bar is embedded in a concrete element
and is subjected to internal forces, the distribution of internal stresses along the element will depend upon the
bond characteristics between concrete and reinforcement and the geometry of the reinforcement. If, for example, a corner portion of a shear stirrup is considered, the average stresses acting on the bent portion of the link ignoring bond stresses can be represented as shown in Figure 1.

Figure1. Average stresses acting on a bent portion of reinforcement embedded in concrete

For the sake of simplicity, a uniform (equivalent hydrostatic) pressure is considered to be exerted by the concrete along the bent portion. The equilibrium of the rigid body may be expressed as

$$\sigma_1 b t = \sigma_2 r b$$

or

$$\sigma_2 = \frac{\sigma_1 t}{r}$$

in which $\sigma_1$ is the tensile stress developed in the straight bar, $\sigma_2$ is the compressive stress applied by the confined concrete perpendicular to the fibres, $r$ is the internal bending radius, and $b$ and $t$ are the width and thickness of the bar, respectively. The combination of $\sigma_1$ and $\sigma_2$ results in the development of a biaxial state of stress acting on the bent portion. One of the most comprehensive failure criteria for composite materials available in the literature is based on the Tsai-Hill theory (Taranu and Isopescu 1996). Tsai applied Hill’s anisotropic plasticity to failure in an orthotropic lamina and developed the failure criteria, later known as Tsai-Hill theory. For a general three dimensional state of stress along the principal axes of anisotropy, the failure surface is described by Eq (4).

$$A (\sigma_1 - \sigma_2)^2 + B (\sigma_2 - \sigma_3)^2 + C (\sigma_3 - \sigma_1)^2 + 2D \tau_{13}^2 + 2E \tau_{31}^2 + 2F \tau_{12}^2 = 1$$

in which coefficients $A$, $B$, $C$, $D$, $E$, and $F$ are determined from yield strength in uniaxial or shear loading tests. For the case of plane stress in the 1-2 plane ($\sigma_3=\sigma_{13}=\sigma_{23}=0$) of a transversely isotropic material, the Tsai-Hill failure surface may be expressed in the form

$$\frac{\sigma_1^2}{\sigma_{1\text{max}}^2} - \frac{\sigma_1 \sigma_2}{\sigma_{1\text{max}} \sigma_{2\text{max}}} + \frac{\sigma_2^2}{\sigma_{2\text{max}}^2} + \frac{\tau_{12}^2}{\tau_{\text{max}}^2} = 1$$

in which $\sigma_{1\text{max}}$ is the longitudinal tensile strength, $\sigma_{2\text{max}}$ is the transverse tensile strength, and $\tau_{\text{max}}$ is the in-plane shear strength. Substituting Eq (2) into Eq (5) for the case illustrated in Figure 1 (+$\sigma_1$ in tension and -$\sigma_2$ in compression), therefore

$$\frac{\sigma_1^2}{\sigma_{1\text{max}}^2} + \frac{\sigma_1}{\sigma_{1\text{max}}} \left(\frac{\sigma_1 t}{r}\right) + \frac{1}{\sigma_{2\text{max}}} \left(\frac{\sigma_1 t}{r}\right)^2 + \frac{\tau_{12}^2}{\tau_{\text{max}}^2} = 1$$

Rearranging, the ratio between the maximum stress that can be sustained along the bend of the composites, $\sigma_1$, and its unidirectional tensile strength, $\sigma_{1\text{max}}$ can be written in the following form (Eq 7).

$$\frac{\sigma_1}{\sigma_{1\text{max}}} = \frac{1}{\sqrt{1 - \phi^2}}$$

in which $\phi=\tau_{12}/\tau_{\text{max}}$ and $\beta=\sigma_{1\text{max}}/\sigma_{2\text{max}}$. Eq (7) was derived assuming a composite bar with a rectangular cross-section. If a round bar is used, the factor $\pi d/4$ replaces the bar thickness, $t$, as shown in Eq (8).

$$\frac{\sigma_1}{\sigma_{1\text{max}}} = \frac{\sqrt{1 - \phi^2}}{1 + \left(\frac{\pi d}{4r}\right)^2 \cdot \beta^2}$$
Figure 2-left shows the effect of shear stress on the bend capacity. It can be seen that the bend capacity can decrease with an increase in the bond stress. As shown in Eqs (7) and 8, the factor $\phi$ is the ratio between the shear stress, $\tau_{12}$, and the maximum shear strength, $\tau_{\text{max}}$. The maximum shear strength is equal to the interlaminar shear strength of the unidirectional composite. In general, $\tau_{\text{max}}$ is much higher than the developed bond stress in concrete and it is not likely that interlaminar shear failure will occur. For a bent unidirectional composite subjected to tensile loading as shown in Figure 1, the value of $\phi$ is very small (not more than 0.2) and can be neglected when determining the bend capacity of the material (Imjai et al. 2004). Figure 2-left shows the capacity that can be developed in the material according to the different value of $\phi$. Another important parameter included in the proposed macromechanical model is the factor $\beta$. This factor is the ratio of the longitudinal tensile, $\sigma_{1\text{max}}$, strength and transverse compressive strength, $\sigma_{2\text{max}}$, of the composite material. Figure 2-right shows the effect of $\beta$ on the bend capacity of the composite. It can be seen that the strength of a bent unidirectional composites can be considered to be equal to its ultimate uniaxial strength only when the composite is characterised by a small value of $\beta$ (around 5) i.e. $\sigma_{2\text{max}}$ value closed to $\sigma_{1\text{max}}$. The value of $\sigma_{2\text{max}}$ for the composite tested during this research project was determined by carrying out a direct compression test. All unidirectional composites were cut into 10x10x10 mm cubes and tested under compressive loading in the direction perpendicular to the fibres. The transverse compressive strength ($\sigma_{2\text{max}}$) of the thermoplastic composite obtained from the tests was found to be 96 MPa (Imjai et al. 2007). The calculated value of $\beta$ for the thermoplastic composite tested in this study was 7.5.

Figure 2. Effects of factor $\phi$ (left) and factor $\beta$ (right) on the bend capacity

Figure 3 shows a comparison of the predicted bend capacity in terms of the average failure stress to the ultimate strength of the straight section ($\sigma_{b,\text{avg}}/\sigma_{1\text{max}}$) calculated by the macromechanical based model. The value of $\sigma_{2\text{max}}$ obtained from the tests was used here. The macromechanical based model adequately captures the variation of the bend capacity with the difference of the bending radius to bar diameter ratio. In some cases, the capacity predicted by the proposed model was found to overestimate the test results especially when compared to specimens P3 (unbonded). This reveals that the development of bond stresses along the bar/concrete interface plays an important role in controlling the maximum capacity that can be carried through the bent portion. Nevertheless, the predictions obtained according to the macromechanical model provide a lower bound solution, particularly when bonded specimens were used.

Figure 3. Analytical predictions of bend capacity of thermoplastic strips according to the JSCE and proposed models
Figure 3 also shows the variation in the strength of the bent specimens according to the equation proposed in the JSCE design recommendations (see Eq. 1), which currently is used in all of the main design recommendations for the design of FRP RC elements (the shaded area demarked by dashed lines). As can be observed, the current design equation does not adequately describe the variation in bend capacity that was observed experimentally. Moreover, it would appear that Eq. 1 could overestimate the bend capacity of the composite strip that was used in this study. Sheata et al. (2000) have also reported a similar tendency for a commercial type of CFRP reinforcement. In the present study, however, acceptable predictions were obtained for those specimens for which a better bond between the composite and the concrete was ensured either by sand coating the strips or through the use of a high strength concrete.

DISCUSSION

Section Effects on the Bent Section

During the material preparation, bending of the thermoplastic strips to different bending radius was made by the application of heat. This process, however, caused the bend cross section to collapse and flatten, inducing the slack of the fibres on the inside of the bend. Moreover, kinking of the fibres located on the inner face of the bend was also observed in the bent specimens tested in this study. The cross-section geometry along the bent portion could affect the nature of stress-strain fields in the bent zone and thus influence the maximum strength that can be developed in the bent portion of FRP unidirectional composites. In Figure 1 and Equation (1), the cross section remained constant. If the actual geometry of the bent portion is taken into account (see Figure 3b), the force equilibrium of the rigid body can be expressed as

$$\sigma_1 \left( \frac{\pi d^2}{4} \right) = \sigma_2 \cdot r \cdot d_b$$

in which \(d\) is the nominal diameter and \(d_b\) is the projected diameter at the bent section of the bar.

The radial stress, \(\sigma_2\), can be expressed as

$$\sigma_2 = \sigma_1 \cdot \frac{\pi}{4r} \cdot \frac{d^2}{d_b}$$

or

$$\sigma_2 = \sigma_1 \cdot \frac{\pi}{4r} \left( \frac{d^2}{d_b} \right) = \frac{\sigma_1 \pi d}{4r} \cdot \psi$$

where \(\psi\) is \(d/d_b\), which is referred to as the section factor \((\psi \leq 1)\). Thus, by substituting Eq (11) into Eq (6), the bend capacity of the material can be rewritten in the form:

$$\frac{\sigma_1}{\sigma_{1_{\text{max}}}} = \frac{1}{\sqrt{1 - \psi^2}} \cdot \frac{\pi d \psi}{4r} \left( \frac{\pi d \psi}{4r} \right)^2 \cdot \beta^2$$

Figure 3. Different geometry at the bent section of FRP unidirectional composites

![Diagram of constant and variable bent sections](image)
If a rectangular cross-section is used, Eq (8) can be rewritten as

\[
\sigma_1 = \frac{\sigma_{1_{\text{max}}}}{\sqrt{1 - \psi^2}} \sqrt{1 + \left(\frac{t \psi}{r}\right) + \left(\frac{t \psi}{r}\right)^2 \beta^2}
\]

(13)

where \(\psi\) is the ratio between the nominal width of the strip to the width at the bent section. As seen in Figure 4, the bend capacity increases as the section factor decreases (i.e. \(d_b > d\)). This is due to the fact that the radial stresses are a function of the geometry at the bent section and reduce when \(d_b > d\). The reduction in \(\sigma_2\) results in higher bend capacity. Hence, more accurate predictions could be obtained by introducing the section factor in the proposed model.

![Figure 4. Effect of the shape factor \(\psi\) on the bent capacity](image)

**Transverse Strength of Unidirectional Composites**

The transverse compressive strength used to validate the value of \(\beta\) in the proposed model was determined from a direct compression test (Imjai et al. 2007). In general, the transverse properties of a composite, including the transverse compressive strength, \(\sigma_{2_{\text{max}}}\), are not readily available, however, as they are not usually provided by the manufacturers. This is because only longitudinal mechanical properties of a composite are of interest for the design of reinforced concrete structures. If the chemical and physical make up of the composite is known, the transverse properties of the composites could be determined on the basis of micromechanical principles and expressed as a function of the compressive strength of the resin matrix, \(f_{mc}\), a stress concentration factor, \(k_\sigma\), and the residual radial stress at the matrix/fibre interface, \(\sigma_{rm}\), as shown in Equation (14) (Taranu and Isopescu 1996). Evaluation of \(\sigma_{2_{\text{max}}}\) based on this approach would require, however, determination of micromechanical properties that are not easily available to designers.

\[
\sigma_{2_{\text{max}}} = \frac{1}{k_\sigma} \left( f_{mc} + \sigma_{rm} \right)
\]

(14)

The stress concentration factor depends on the relative properties of the constituents and their volume fraction in the composite system and can be expressed in the form

\[
k_\sigma = \frac{1 - V_f \left(1 - E_m/E_f\right)}{1 - \left(4V_f/\pi\right)^{1/2} \left(1 - E_m/E_f\right)}
\]

(15)

in which, \(V_f\) is the fibre volume fraction, \(E_f\) is the elastic modulus of the fibres, and \(E_m\) is the elastic modulus of the resin matrix.
CONCLUDING REMARKS

Based on the analytical work presented here, the following conclusions can be drawn:

- The capacity of curved FRP composites appeared to be mainly a function of the geometry of the test specimens, namely the bending radius.
- The capacity of the bent specimens does not seem to vary linearly with the $r/d$ ratio, as defined in the JSCE equation, and does not appear to be solely a function of the bend geometry. Rather, bond characteristics appeared to be important in controlling the development of stresses along the embedded portion of the composite and in dictating its ultimate behaviour.
- In a significant number of cases, the equation included in the current design recommendations for concrete structures reinforced with FRP was found to overestimate the bend capacity of the composite strip used in this study.
- The macromechanical based failure model that is proposed, adequately capture strength degradation due to the change in geometry of the FRP bar.
- The variability of the cross section along the bend can be taken into account by introducing a section factor into the proposed model.

ACKNOWLEDGMENTS

The author wishes to acknowledge the financial assistance of the European Union for the Marie Curie Research Training Network En-Core, and the CRAFT RTD project CurvedNFR.

REFERENCES


