Finite element model for intermediate crack debonding in RC beams strengthened with externally bonded FRP reinforcement

G.M. Chen¹, J.G. Teng¹, J.F. Chen² & O.A. Rosenboom¹
¹Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, China
²Institute for Infrastructure and Environment, The University of Edinburgh, UK

ABSTRACT: Intermediate crack induced debonding (IC debonding) is a common failure mode of RC beams strengthened with externally bonded FRP reinforcement. Although extensive research has been carried out on IC debonding, much work is still needed to develop a better understanding of the failure mode and a more reliable design method. This paper first examines the limitations of existing finite element (FE) approaches for modelling IC debonding and then proposes an improved approach. In the proposed approach, concrete cracking is modelled using the crack band model while the interfacial behaviour (between concrete and both internal steel and external FRP reinforcements) is captured using interfacial elements with appropriate bond-slip properties. These features enable the close prediction of both the spacing and the width of major cracks in the concrete, which have an important bearing on the IC debonding failure process. The capability and accuracy of the proposed approach are illustrated through a comparison of the present FE predictions with the test results for an example beam.

1 INTRODUCTION

It is now a common practice to bond fiber-reinforced polymer (FRP) plates to the tension face of reinforced concrete (RC) beams to increase their flexural load-carrying capacity. Such strengthened beams often fail by debonding in two types of modes (e.g. Teng and Chen 2007): (a) intermediate crack (IC) induced debonding in which debonding is initiated at a major flexural crack and propagates towards a plate end; and (b) plate end debonding including a number of distinct modes in which debonding initiates at the plate end. Debonding failures involve complex mechanisms and are still a topic of intensive current research. The paper first examines the limitations of existing finite element (FE) approaches for modelling IC debonding and then proposes an improved FE approach. An example is presented to illustrate the capability and accuracy of the proposed approach. This work is reported in more detail in Chen et al. (2008).

2 MODELLING OF CONCRETE CRACKING

Extensive research has been conducted on the numerical modelling of concrete cracking; a summary of the existing knowledge based on ACI 446.3R (1997), Bazant and Planas (1998) and de Borst et al. (2004) is presented in this section to provide the background to the present work. Concrete cracking may be modelled using either the discrete crack model or the smeared crack model (ACI 446.3R 1997). The former is also known as the fictitious crack model (FCM), which simulates a crack as a geometrical identity so the discontinuities arising from cracking are physically modelled. By contrast, the smeared crack model treats the cracked concrete as a continuum. It captures the deterioration process in cracked concrete through a constitutive relationship and hence smears cracks over the continuum.

When the discrete crack model is implemented in an FE analysis, the cracks are commonly defined along element boundaries. This inevitably introduces mesh bias (ACI 446.3R 1997). At-
tempts have been made to solve this problem by developing FE codes with automatic re-meshing algorithms (e.g. Yang et al. 2003), but overcoming computational difficulties associated with topology changes due to re-meshing remains a challenge (de Borst et al. 2004).

Smeared crack models may be divided into two types: the fixed smeared crack model and the rotating smeared crack model; they differ in the assumption made for the direction of crack propagation and in the definition adopted for the shear retention factor. A drawback of the smeared crack model is that it produces the phenomenon of “strain localization”, leading to zero energy consumption during crack propagation when the element size approaches zero; as a result, the solution is not mesh objective. Various mathematical devices which are commonly called “localization limiters” have been proposed to overcome the mesh non-objectivity problem. One of the successful localization limiters is the crack band model, which relates the element size to the constitutive law of the concrete so that the fracture energy is independent of element size. Other localization limiters include the non-local continuum model and the gradient model (ACI 446.3R 1997). When a localization limiter such as the crack band model is employed, the discrete crack model and the smeared crack model yield about the same results if the crack opening displacement $w$ in the discrete crack is taken as the cracking strain $\varepsilon_{cr}$ accumulated over the width $h_c$ of the crack band in a smeared crack model (Bazant and Planas 1998):

$$w = \int_{h_c}^{\varepsilon_{cr}} dh$$

3 EXISTING FE STUDIES ON DEBONDING FAILURE OF FRP-PLATED RC BEAMS

Both the discrete crack and the smeared crack models have been employed in modelling debonding failures of FRP-plated RC beams. A brief review of the existing studies with particular attention to the crack modelling approaches is presented below.

3.1 Discrete crack approach

Yang et al. (2003) modelled the initiation of cracking as the complete separation of two nodes at the same location without any post-cracking traction between them in order to simulate debonding failures of FRP-plated RC beams within the context of linear elastic fracture mechanics (LEFM). They used an automatic re-meshing algorithm with limited success. The LEFM, which treats all constituent materials as being linear elastic, is simplistic and incapable of predicting the whole loading process. Although they later implemented the FCM to simulate the fracture process zone (FPZ), numerical challenges arising from re-meshing, especially when a complex pattern of non-parallel multiple cracks exists, remain to be overcome.

Niu and Wu (2005) also used the discrete crack approach to model debonding failures of FRP-strengthened RC beams. In their study, flexural cracks were pre-defined using vertically oriented and evenly distributed interfacial elements. As crack spacing has a significant effect on IC debonding as demonstrated by this and other studies (Teng et al. 2006), numerical results from pre-defined cracks are useful to understand the effect of crack spacing but are clearly not predictive. Similarly, Niu et al. (2006) used pre-defined diagonal cracks to simulate CDC debonding failures in FRP-strengthened RC beams.

Kishi et al. (2005) adopted a more sophisticated approach in which the discrete crack model was combined with the smeared crack model; the former was employed to simulate geometrical discontinuities such as of the development of dominant cracks, slipping of axial rebars, and debonding of FRP plates while the latter was used to model minor cracks. In their method, the discrete cracks were also pre-defined. The crack band model was used in association with the smeared crack model. In the discrete crack model, the tensile behaviour of concrete was assumed to be linear elastic brittle.

3.2 Smeared crack approach

The smeared crack approach has been adopted in several studies for modelling FRP-plated RC beams (e.g. Wong and Vecchio 2003, Neale et al. 2006, Lu et al. 2007). Wong and Vecchio (2003) used an FE program developed based on the Modified Compression Field Theory
where a tension stiffening model was used to describe the bond effect between reinforcing bars and cracked concrete. However, the tension stiffening model is prone to spreading crack formation over a finite region and is thus incapable of depicting localized crack propagation. Furthermore, the tension stiffening equation implemented in the FE program was based on the test results of 30 panel elements with dense steel reinforcement; this equation is thus not suitable for RC beams where the concrete at some distance away from the tension bars is poorly reinforced (Bentz 2005). The use of a tension stiffening model may also lead to mesh dependency.

In Neale et al. (2006) and Lu et al. (2007), the steel rebars were tied to the concrete so no interfacial slips between them were considered. According to Rots (1988, 1989), this assumption may lead to inaccurate prediction of the spacing and the width of the critical cracks, which in turn has a significant effect on the predicted debonding behaviour. Furthermore, Neale et al. (2006) did not explain how they dealt with the mesh-dependent feature of the smeared crack method.

4 PROPOSED FE MODEL FOR IC DEBONDING
As discussed earlier, the spacing and width of concrete cracks have a significant effect on IC debonding behaviour in FRP-plated RC beams. In an FE model, the main factors that influence the predicted spacing and width of concrete cracks include the details of the concrete crack model, the bond-slip properties between concrete and internal steel reinforcement, and the bond-slip properties between concrete and external FRP reinforcement. In the present study, efforts were made to develop an FE model which takes these factors into proper account. The proposed model is a two-dimensional FE model and has been implemented in ABAQUS (Version 6.5). It is based on the smeared cracked model so that the crack paths need not be pre-defined and employs the crack band model to overcome the mesh sensitivity problem associated with the conventional smeared crack model. The proposed model is summarised as follows.

4.1 Modelling of concrete
The concrete was modelled using the plane stress element CPS4 in ABAQUS (Version 6.5) incorporating the crack band model for modelling the cracking behaviour. The crack band model was defined using the concrete damaged plasticity model in ABAQUS. For concrete under uniaxial compression, Chen’s (1982) stress-strain relationship was adopted:

\[ \sigma = \frac{a \varepsilon}{1 + \left( \frac{a \varepsilon}{\sigma_p} - 2 \frac{\varepsilon}{\varepsilon_p} \right) + \left( \frac{\varepsilon}{\varepsilon_p} \right)^2} \]  

(2)

in which \( \sigma \) and \( \varepsilon \) are the principal stress and strain respectively, \( \sigma_p \) and \( \varepsilon_p \) are respectively the experimentally determined maximum principal stress and the corresponding strain, and \( a \) is an experimentally determined coefficient representing the initial tangent modulus. In this study, \( a \) was set to be equal to the elastic modulus of the concrete \( E_c \) and the ACI 318 (2002) equation was used to estimate \( E_c \) from the cylinder compressive strength (i.e. \( E_c = 4730 \sqrt{f'_c} \) in MPa). \( \sigma_p \) and \( \varepsilon_p \) were set to be equal to \( f'_c \) and 0.002 respectively.

For concrete under uniaxial tension, the tension softening curve of Hordijk (1991) which was derived based on an extensive series of tensile tests of concrete was employed:

\[ \frac{\sigma_t}{f_t} = \left[ 1 + \left( \frac{w_t}{w_{cr}} \right)^{c_1} \right] \exp \left( -c_2 \frac{w_t}{w_{cr}} \right) - \frac{w_t}{w_{cr}} \left( 1 + c_1 \right) e^{-(c_2)} \]  

(3)

\[ w_{cr} = 5.14 \frac{G_F}{f_t} \]  

(4)

where \( w_t \) is the crack opening displacement, \( w_{cr} \) is the crack opening displacement at the complete release of stress or fracture energy, \( \sigma_t \) is the tensile stress normal to the crack direction, and \( f_t \) is the concrete tensile strength under uniaxial tension. \( G_F \) is the fracture energy required to create a stress-free crack over a unit area, and \( c_1=3.0 \) and \( c_2=6.93 \) are constants determined.
from tensile tests of concrete. In FE modeling, if no test data are available, \( f_t \) and \( G_f \) may be estimated from the following CEB-FIP (1993) equations:

\[
f_t = 1.4 \left( \frac{f_c^e - 8}{10} \right)^{0.7}
\]

\[
G_f = (0.0469d_a^2 - 0.5d_a + 26f_c^e)^{0.7}
\]

where \( d_a \) is the maximum aggregate size. In the present paper, it is assumed that \( d_a = 20 \) mm.

The stress-displacement curve defined by Equations 3-6 can be transformed into a stress-strain curve according to the crack band model as depicted by Equation 1. In ABAQUS, the crack band width \( h_c \) is defined as the characteristic crack length of an element. In the present study, the recommendation for estimating crack band widths made by Rots (1988) was followed. For instance, the characteristic crack length of a plane stress four-node square element was taken to be \( \sqrt{2} e \), where \( e \) is the length of the square element.

In the present study, it was assumed that the Poisson’s ratio \( \nu = 0.2 \) and the dilation angle \( \varphi = 35^\circ \). Numerical results not given in this paper showed that both parameters have little effect on the predictions if failure is not controlled by the compressive crushing of concrete. A user-defined damage curve was employed to account for the progressive degradation of the shear resistance of concrete as the cracks widen (Chen et al. 2008).

4.2 Modelling of steel, FRP and bond behaviour

Both the steel and the FRP reinforcements were modelled using truss elements. The steel reinforcement was assumed to be elastic-perfectly plastic and the FRP reinforcement was assumed to be linear elastic brittle.

The bond behaviour between internal steel reinforcement (both longitudinal rebars and stirrups) and concrete was modelled using the nonlinear spring element Spring 2 in ABAQUS. Two springs were defined between every pair of steel rebar and concrete nodes at the same location to model the shear and normal bond responses respectively. The normal spring was assumed to be linear elastic with a large stiffness so no failure and little separation or overlapping between the two connected nodes can occur. The bond force \( F_{b,s} \) in a shear spring was determined using the CEB-FIP (1993) bond-slip model as:

\[
F_{b,s} = n \times \pi \times D \times l_i \times \tau_s
\]

where \( n \) is the number of steel rebars represented by the truss element, \( D \) is the diameter of the steel rebars and \( l_i \) is the average length of the two adjacent truss elements and \( \tau_s \) is the bond shear stress. The bond-slip behaviour was assumed to be fully reversible, allowing the use of the spring element Spring 2 rather than the interfacial element COH2D4 available in ABAQUS, which should not result in any significant difference during the loading process until the peak load is reached as the concrete-to-steel rebar interfaces are unlikely to experience any unloading during this process. Both types of elements may be referred to as interfacial elements in general.

The bond behaviour between FRP and concrete was modelled using the interfacial element COH2D4 in ABAQUS. Along the interface, the properties of the interfacial element were defined from the simplified bond-slip model for FRP externally bonded to concrete proposed by Lu et al. (2005). Linear damage was assumed for the local bond-slip relationship after the interface enters the softening range. Normal to the interface, the element was assumed to behave linear elastically with the normal stiffness estimated from the stiffness of the adhesive layer.

5 NUMERICAL RESULTS

A number of laboratory tests on IC debonding failures of FRP-plated RC beams from various sources have been successfully simulated using the proposed FE approach. The test specimens covered a large range of the following parameters: (a) beam dimensions; (b) material properties; and (c) strengthening configurations. The FE model was also employed to predict the crack
spacings and widths of several RC beams with or without FRP strengthening, which showed that the present FE model is capable of accurate predictions of cracking behaviour. Due to space limitation, only specimen D2 tested by Brena et al. (2003) is presented herein as an example beam to illustrate the accuracy of the FE model while additional verifications of the model are available in Chen et al. (2008). The example beam was chosen because the test data of this beam were clearly reported by the authors. The beam was 368 mm in width and 406 mm in depth with a span of 3,000 mm. It was tested under four-point bending with a shear span of 1,220 mm. The cylinder compressive strength of concrete was 37.2 MPa. The beam was reinforced with 2φ16 tension steel bars and 2φ9.5 compression steel bars. The yield strength of both compression and tension steel bars $f_y = 440$ MPa. Within the shear spans, the beam was reinforced with double-leg φ7 steel stirrups. The yield strength of the stirrups $f_{ys} = 596$ MPa. The beam was flexurally strengthened by externally bonding a pultruded CFRP plate with a width of 50 mm, a thickness of 1.19 mm, an elastic modulus of 155GPa and a tensile strength of 2,400 MPa. Brena et al. (2003) reported that this beam failed by IC debonding which initiated in the region under one of the loading points. Only half of the specimen was modelled taking advantage of symmetry. A convergence study showed that the predicted load-displacement curves change very little when the maximum concrete element size is less than 30mm. A maximum element size of 15 mm was thus used for the concrete in the example beam. Matching element sizes were chosen to represent the FRP and the steel reinforcements. The numerical results are given in Figures 1-3.

![Figure 1](image1.png)  ![Figure 2](image2.png)  ![Figure 3](image3.png)  ![Figure 4](image4.png)

Figure 1. Load versus displacement.  Figure 2. Strain distributions in FRP.  Figure 3. Distributions of interfacial shear stress.  Figure 4. Crack pattern at debonding failure.

Figure 1 shows that the predicted load-displacement curve is in close agreement with the test curve. The point of IC debonding on the test curve is indicated in Figure 1 according to the observation of Brena et al. (2003). Figure 2 shows the distributions of axial strain in the FRP plate for the following states: (a) before cracking of the concrete, (b) before yielding of the longitudinal steel rebars, (c) after yielding of steel rebars, (d) at the beginning of FRP debonding, and (e)
during debonding propagation. The mid-span deflections of these states are given in the figure. Note that the predicted maximum FRP strain during the debonding process is $5500\mu\varepsilon$. Only one strain gauge was installed on the test beam at a distance of 1142 mm from the plate end. The predicted FRP debonding strain at the location of this strain gauge is about $5300\mu\varepsilon$, which is slightly larger than the test value of $4800\mu\varepsilon$. Given the non-uniform strain distribution, these strain values also indicate a close agreement between the test and FE results. The distributions of the interfacial shear stress for the same five states are shown in Figure 3 and are seen to be highly non-uniform. The numerical results show that FRP debonding starts from the major flexural crack under the loading point when the mid-span deflection is about 12 mm. When the mid-span deflection reaches 12.8 mm, debonding has propagated by about 200 mm towards the plate end, as evidenced by the part of the curve with a constant FRP strain (Figure 2) and zero interfacial shear stress (Figure 3). The predicted cracking pattern at debonding failure (i.e. with a mid-span deflection of 12.8 mm) is shown in Figure 4. The numerical results given in Figures 1-4 clearly demonstrate that the proposed FE approach is capable of correctly predicting the failure mode (i.e. IC debonding). Furthermore, it provides reasonably accurate predictions of the ultimate load and the maximum value of the axial strain in the FRP plate at debonding failure.

6 CONCLUSIONS
Following a critical examination of the limitations of existing FE approaches for the prediction of debonding failures in FRP-strengthened RC beams, this paper has proposed an improved FE method. In the proposed approach, concrete cracking is modelled using the crack band model while the interfacial behaviour (between concrete and both internal steel and external FRP reinforcements) is captured using interfacial elements with appropriate bond-slip properties. The numerical predictions of a test beam have shown that the proposed approach can accurately simulate the full-range behaviour of an IC debonding failure.

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8 REFERENCES
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