AN INVESTIGATION ON THE STRESS TRANSFER IN CONCRETE BEAMS BONDED WITH FRP PLATES

Jianqiao Ye ¹ and Jian Yang ²

¹ School of the Civil Engineering, The University of Leeds, UK
² School of Engineering, University of Birmingham, Edgbaston, Birmingham, B15 2TT

ABSTRACT

Strengthening of concrete beams by the external plate bonding technique has become an increasingly important research area over the last two decades, especially with the increasing applications of the advanced fibre reinforced composites. Unlike the un-strengthened beams, the composite structure shows an undesirable failure of plate debonding. It has been well recognized that the central to this failure mode is the stress concentration at the plate end. This paper reports a novel interfacial stress formulation for the FRP bonded RC beams subjected to arbitrary loadings. It is featured as a compact formulation with improved accuracy. Numerical solutions are given to compare the simplified solutions with those obtained from more rigorous formulations and alternative solutions

KEYWORDS

FRP, RC beams, strengthening, interfacial stresses, stress concentration.

INTRODUCTION

As a nation’s infrastructure ages, one of the major challenges the construction industry has to face is that the number of deficient structures continues to grow. The applications of using externally bonded steel plates or fiber reinforced polymer (FRP) laminates to reinforce concrete (RC) structures have shown that the technique is sound and efficient and offers a practical solution to this pressing problem. It has been realized that central to the reinforcement effect of externally bonded concrete structures is the transferring of stresses from the concrete to the external reinforcement, which causes the undesirable premature and brittle failure modes, such as debonding initiated from the ends of the bonded plates (Teng, et al., 2002). A good understanding of this problem is thus important for the development of suitable strength models. Apart from the experimental investigations carried out in the last two decades (e.g., Bonacci and Maalej, 2001), analytical solutions have been played an important role in seeking a fundamental understanding of the problem. These works include the shear-lag approach by Triantafillou and Deskovic (1991) and Ye (2001); the staged analysis approach by Roberts and Haji-Kazemi (1989); the deformation compatibility-based approach by Malek et al. (1998) and Smith and Teng (2001) and Rabinovich and Frostig’s (2000) high-order solution.

This paper reports a novel interfacial stress formulation for the FRP bonded RC beams subjected to arbitrary loadings. The formulation is established on the basis of the simplified form of a rigorous elasticity solution proposed by Yang and Ye (2005) and the introduction of the concept of bond length. It is featured as a compact formulation with improved accuracy. The paper also validates the simplified formulation through comparisons with the rigorous solutions and solutions proposed by other researchers.

SIMPLIFIED INTERFACIAL STRESSES UNDER END MOMENT

Basic rationale for the simplified solution and the derivation procedures

Yang and Ye (2005) proposed a rigorous elastic solution for a bonded RC beam subjected to uniformly distributed load (UDL) and a pair of equal bending moments. Yang and Ye (2004) has numerically verified the following observations:

1. The main cause of stress concentration occurring near the plate ends is the bending moment acting on the cross section at the plate end;
The transversely applied load and the induced internal shear forces have a negligible impact on the stress concentration at the plate end. Their contributions to the interfacial shear stresses can be computed based on the classic beam theory.

Based on the above observations, the significant parts in the interfacial stresses are the stresses caused by the bending moments $M_l$ on the cross sections at the plate ends, which equal the interfacial stresses in the strengthened beam under end moments (see Fig. 1).

![Figure 1. A plated beam under end bending moment](image)

**Simplified shear stress solution**

The same idealised model used in Yang and Ye (2005) is used here, in which we let $q = 0$, and $M_0 = M_l$ (see Figure 1). The simplified solution is based on omitting some relatively small elements from the rigorous solution, which was formulated from two-dimensional elasticity. These small terms include the thickness ratio of both FRP plate and adhesive layer to concrete and terms that are numerically small in practical engineering applications. Other simplifications include expressing the hyperbolic functions approximately in terms of exponential functions, such as

$$\frac{\sinh \gamma_1 x}{\sinh \gamma_1 l} = e^{\gamma_1 (l-x)} \frac{e^{2\gamma_1 x} - 1}{e^{2\gamma_1 l} - 1} = e^{\gamma_1 (l-x)} \frac{1 - e^{-2\gamma_1 x}}{e^{2\gamma_1 (l-x)} - e^{-2\gamma_1 x}} \approx e^{\gamma_1 (l-x)}$$

Due to the aforementioned observations, a single solution is given for the interfacial shear stresses at both adhesive-concrete (AC) and the plate-adhesive (PA) interfaces, which is

$$\sigma_{xy} = \frac{h_0 h_1^{[1]} \gamma_1 \gamma_2 \left[ e^{\gamma_1 (x-l)} - e^{\gamma_2 (x-l)} \right]}{I \left( \gamma_1 - \gamma_2 \right)} M_l$$

(2)

The maximum shear stress occurs at

$$x^* = \frac{\ln \gamma_1 - \ln \gamma_2}{\gamma_1 - \gamma_2}$$

(3)

with a value of

$$\sigma_{xy}^{\text{max}} = -\frac{h_0 h_1^{[1]} \gamma_1}{I} \left( \frac{\gamma_1}{\gamma_2} \right)^{\gamma_1 - \gamma_2} M_l$$

(4)

In the above equations, $h_1^{[1]}$, $E_1^{[1]}$, $E_y^{[1]}$, $G_y^{[1]}$ and $v_y^{[1]}$, denote thickness, Young’s modulus, shear modulus and Poisson’s ratio, respectively, of the bonded plate. Replacing [1] by either [2] or [3] yields the parameters for the adhesive layer and the concrete, respectively, of the beam. $h_0$ is the distance from the neutral axis to the lower edge of the cross section transformed to the plate material,

$$h_0 = \frac{b h_1^{[1]} + b E_1^{[2]} h_2^{[2]} \left( h_1^{[1]} + h_1^{[2]} \right) + B E_1^{[3]} h_1^{[1]} h_2^{[1]} + h_2^{[3]} h_1^{[2]}}{E_1^{[2]} h_2^{[2]} + B E_1^{[3]} h_1^{[1]} h_2^{[1]}}$$

(5.1)

$I$ is the second moment of area of the same transferred cross section about its neutral axis and is as follows

$$I = \frac{bh_2^{[2]} h_1^{[1]} + h_2^{[2]} h_1^{[2]} + B E_1^{[3]} h_1^{[1]} h_2^{[1]}}{E_1^{[2]} h_2^{[2]} + B E_1^{[3]} h_1^{[1]} h_2^{[1]}}$$
I = \frac{1}{12} \left[ b(h^{(1)})^3 + \frac{bE_s^{(2)}}{E_y^{(1)}}(h^{(2)})^3 + \frac{BE_s^{(3)}}{E_y^{(1)}}(h^{(3)})^3 \right] + bh^{(1)} \left( \frac{h^{(1)}}{2} - h_0 \right)^2 + \frac{bE_s^{(2)}}{E_y^{(1)}} h^{(2)} \left( \frac{h^{(1)}}{2} + \frac{h^{(2)}}{2} - h_0 \right)^2 + \frac{BE_s^{(3)}}{E_y^{(1)}} h^{(3)} \left( \frac{h^{(1)}}{2} + \frac{h^{(2)}}{2} + \frac{h^{(3)}}{2} - h_0 \right)^2

(5.2)

\gamma_1 \text{ and } \gamma_2 \text{ can be computed from }

\gamma_1 = \sqrt{\frac{S_2 + \sqrt{S_2^2 - 2S_3S_1}}{2S_3}}

(5.3)

\gamma_2 = \sqrt{\frac{S_2 - \sqrt{S_2^2 - 2S_3S_1}}{2S_3}}

(5.4)

where

\begin{align*}
S_1 &= b \left[ \frac{4}{E_s^{(3)b}h^{(3)}} + \frac{1}{E_s^{(1)b}h^{(1)}} \right] \\
S_2 &= \frac{h^{(3)}b}{15G_y^{(2)}B + 2G_y^{(2)} + 6G_y^{(1)}} \\
S_3 &= \frac{h^{(2)}h^{(3)}}{E_s^{(2)}} \left[ \frac{(h^{(1)})^2}{4} + \frac{(h^{(2)})^2}{6} \right]
\end{align*}

(5.5) (5.6) (5.7)

Simplified normal stress solution

On the basis of the rigorous solution and by introducing the above mentioned simplifications, the simplified normal stresses at PA and AC interfaces can be expressed, respectively, as

\begin{align*}
\sigma_y^{PA} &= \sigma_y^{\text{Mt}} - \frac{h_0h^{(1)}h^{(2)}}{2I} \left\{ M_1 \gamma_1 \gamma_2 \left[ \gamma_1 \epsilon_y^{(x-l)} - \gamma_2 \epsilon_y^{(x)} \right] \right\} - q(x) \\
\sigma_y^{AC} &= \sigma_y^{\text{Mt}} + \frac{h_0h^{(1)}h^{(2)}}{2I} \left\{ M_1 \gamma_1 \gamma_2 \left[ \gamma_1 \epsilon_y^{(x)} - \gamma_2 \epsilon_y^{(x-l)} \right] \right\} - q(x)
\end{align*}

(6.1) (6.2)

in which, \( \sigma_y^{\text{Mt}} \) is the normal stress on the middle section of the adhesive layer and can be calculated by the following equation

\[ \sigma_y^{\text{Mt}} = K_1 \sin \eta_1 (x - l) e^{\eta_1(x-l)} + K_2 \cos \eta_1 (x - l) e^{\eta_1(x-l)} + K_3 e^{\eta_1(x-l)} + K_4 e^{\eta_1(x-l)} \]

(7)

In equation (7), \( K_1, K_2, K_3 \) and \( K_4 \) are four constants that are material and geometry dependent. The expressions of these constants are not given here and can be found from Yang (2005). \( \eta_1 \) and \( \eta_2 \) are

\begin{align*}
\eta_1 &= \sqrt{\frac{2S_2S_2 + S_3}{4S_3}}; \quad \eta_2 = \sqrt{\frac{2S_2S_3 - S_1}{4S_3}}; \\
S_1 &= \frac{12b^{(1)}}{E_s^{(1)}(h^{(1)})^3} + \frac{12b^{(1)}}{b^{(3)}E_s^{(3)}(h^{(3)})^3} ; \quad S_2 = \frac{3b^{(2)}}{5G_y^{(2)}h^{(1)}} + \frac{3b^{(2)}}{5b^{(3)}G_y^{(2)}h^{(3)}} ; \quad S_3 = \frac{h^{(2)}h^{(3)}}{2E_s^{(2)}}
\end{align*}

(8a-e)

The maximum normal stresses occur at \( x=l \) and are, respectively,

\[ \left( \sigma_y^{\text{Mt}} \right)_{\text{max}} = K_2 + K_3 + K_4 \]

(9)
\[(\sigma_p^{\text{int}})_{\text{max}} = K_2 + K_3 + K_4 - \frac{h_k h_1 h_2}{2 I_0} [M_0 \gamma_1 \gamma_2 - q_i] \] (10)

\[(\sigma_p^{\text{ext}})_{\text{max}} = K_2 + K_3 + K_4 + \frac{h_k h_1 h_2}{2 I_0} [M_0 \gamma_1 \gamma_2 - q_i] \] (11)

where \(q_i\) is the applied load at plate ends.

It is apparent that the simplified formulations of both interfacial shear and normal stresses are mathematically simple and are suitable for engineering applications with the aid of a portable calculator, though further simplifications may still be possible.

SIMPLIFIED INTERFACIAL STRESSES UNDER ARBITRARY LOADS

Concepts of development length

As mentioned in the preceding section, the stress concentration is attributed to the induced bending moments in the cross sections at the plates under the applied loads. Previous studies, e.g. Yang et al. (2004), have revealed that the stress concentration occurs in a local region near the plate ends. In this region, the bonded plate is developing the stress by the bonding actions through the adhesive up to its full composite commission. Thus, this region is called transition zone. Beyond the transition zone, the full composite action takes place among the three material phases, towards which classic laminate beam theory (CLBT) is applicable. The length of the transition zone is called development length, and it is defined as the distance from the end of the plate to the point where full composite action occurs (Nguyen et al. 2001). Brosens and Gemert (1998) defined the development length as the length, at which the bonded plate needs to attain 97% of the maximum force. Based on this definition together with Equation (2), we can calculate the development length \(l'\) using the following equation

\[
\int_{l'-l} \sigma_{\text{eff}}^2 (x)dx = 0.97 \Sigma_p
\] (12)

where \(\Sigma_p\) is the resultant force in the cross section of the bonded plate. Hence

\[
l' = \frac{\ln(\gamma_1) - \ln(\gamma_1 - \gamma_2 + 3.55)}{\gamma_2}
\] (13)

In practical engineering, the bonded plates are usually long enough to develop their full stresses, i.e., the bonded length \(l > l'\).

The calculation of the interfacial stresses under arbitrary loads

The concept of transition zone and development length also provides a means to extend the present simplified solution to those subjected to arbitrary loadings. The bending moment experienced by the cross sections at the plate ends only induce the stress concentration within the transition zones. The solutions of Equations (2) and (7) can be utilized to calculate the stresses within the transition zones. Hence, the interfacial stresses for a plated beam subjected to arbitrary loadings can be calculated by the following steps:

1. Calculate the interfacial shear stress using the CLBT;
2. Calculate the development length for the given beam configuration;
3. Calculate the bending moment in the cross sections at both plate ends;
4. Calculate the interfacial shear and normal stresses using Equations (2) and (7) for the transition zones at both ends using the computed bending moment from (3);
5. The final interfacial normal stresses in the two transition zones are from (4) and zero in the remaining regions;
6. The final interfacial shear stresses in the two transition zones are those from (4) superposed by those from (1) and in the remaining regions are those from (1) only.

NUMERICAL EXAMPLES

To validate the simplified solution, a simply supported RC beam bonded with a CFRP plate and subjected to a single point load of 150 kN at 600 mm away from the left end of the CFRP plate. The dimensions of the RC
beam are $L = 3000\text{mm}$, $b^{[3]} = 200\text{mm}$, and $h^{[3]} = 300\text{mm}$. The RC beam is assumed to be isotropic with $E_x^{[3]} = E_y^{[3]} = 30\text{GPa}$ and $\nu_{xy}^{[3]} = 0.17$. The dimensions and properties of the FRP plate are $l = 2400\text{mm}$, $b^{[1]} = 200\text{mm}$, $h^{[1]} = 4.0\text{mm}$, $E_x^{[1]} = 100\text{GPa}$ and $G_{xy}^{[1]} = 5\text{GPa}$. Those for the adhesive layer are $b^{[2]} = 200\text{mm}$, $h^{[2]} = 2.0\text{mm}$, $E_x^{[2]} = E_y^{[2]} = 2\text{GPa}$ and $\nu_{xy}^{[2]} = 0.35$.

The interfacial stresses are calculated using the simplified solution and compared with a closed form high order solution of Yang et al (2004) and deformation compatibility-based solution of Smith Teng (2001) for the beam. In Figure 2, the normal stress distributions in the AC interface and the MA section are presented. The normal stress in the AC interfaces is predicted by the present simplified solution and Yang et al’s (2004) solution. Clearly, Yang et al.’s solution leads to a much higher peak stress. The stress in the MA section is given by the present simplified solution, which agrees well with the results predicted by Smith and Teng (2001).

In Figure 3, the three curves are, respectively, related to the present simplified solution, Yang et al’s (2004) solution and Smith and Teng’s (2001) approximate solution. The main figure details the stress distributions in the region near the plate ends and the small figure in the upper right area shows the overall distribution. Smith and Teng (2001) predicted a higher peak shear stress, while the other two solutions are reasonably close with comparable peak stresses. It is noted that the present simplified solution leads to a peak position closer to the plate end.

![Figure 2. Simplified approximate solutions of normalized interfacial normal stresses](image)

![Figure 3. Simplified approximate solutions of normalized interfacial shear stresses](image)
CONCLUSION

A simplified approximate solution of the interfacial stresses in FRP plates bonded beams has been presented in this paper. The simple formulations were proposed by omitting some numerically small terms from the rigorous solutions in Yang (2005).

Comparisons have been made to validate the simplified solutions. Good agreement has been observed for both shear and normal stresses along the interfaces.

Compared to other simple solutions, such as a shear-lag type solution, the present solution can not only exhibits the stress concentration near plate ends, but also satisfy traction free condition at the ends.

The simple formulations can be adopted to compute the stresses by using a spreadsheet based software or even a programmable hand calculator.

Further numerical validation and calibration through laboratory tests are needed before the simple solution can be used in practical engineering designs.

REFERENCE