This paper presents the flexural behavior of ballastless track slabs reinforced with BFRP and SFCB and prestressed with steel tendons. Three dimensional nonlinear finite-element analysis and iterative sectional analysis were conducted to predict the behavior of the prestressed members, including experimental validation. The slabs were simply supported and monotonically loaded until failure occurs. Comparisons between the experimental results and the predictions from theoretical analysis showed that the models we adopted could adequately predict the load carrying capacity and deflection of the slabs. The prestressing levels were the main parameters investigated. The applicability of code provisions and existing predictive equations were examined.

1 INTRODUCTION

Compared with a ballasted track slab system, a ballastless track slab system exhibits better performance in terms of stability, durability, and maintenance in high-speed railways [1]. In the ballastless track system, a resonant jointless track circuit is commonly used for high-speed railways in China and Korea. The closed-loop circuit, which consists of longitudinal and transverse reinforcements in the ballastless track slabs, significantly reduces the transmission performance of the track circuit because a mutual inductance is created between the ballastless track slabs and the electric current in the rail. Thus, the effect of the reinforcements on the electrical properties (the rail resistance and inductance) of the rail poses a significant problem [2-5].

Fiber-reinforced polymer composite bars (FRPs) and steel-fiber-reinforced polymer composite bars (SFCBs), allow improved insulation performance and have better anti-corrosion properties, higher tensile strength, and a lighter weight compared with ordinary steel bars [6]. Many scholars focused on the test of ballastless track slab system, but little for theory analysis of FRP reinforced ballastless track slab. In this paper, based on the experimental results of ordinary steel, SFCB and BFRP reinforced ballastless track slabs, an elaborate finite element model is proposed for simulating the nonlinear behavior of ballastless track slabs using the Ansys 13.0. The procedures for element selection and material modeling are presented in detail using an elastic-plastic constitutive model. The numerical model considering both the material and the geometric nonlinearities can fully reflect the complex characteristics such as load-deflection, the strain changes, etc. and the applicability of code provisions and existing predictive equations are examined.
2 SIMULATED EXPERIMENTAL PROGRAM

Yang [4] tested the structural performance of ballastless track slabs reinforced with BFRP, steel and SFCB respectively. An RC slab consisting of eight No. 8 steel bars served as the control specimen. For the S6B27 reinforced slabs and BFRP reinforced slabs, the transverse reinforcements consisted of eight No. 11 S6B27s and eight No. 16 BFRPs, respectively. In addition, six prestressed steel bars were located in the cross section of each slab and had a nominal diameter of 10 mm, a yield stress of 1,449 MPa, and a tensile strength of 1,725 MPa. The total prestress force was 409 kN. The dimensions chosen for the study of the structural performance of the slabs were 1,275 mm long × 650 mm wide × 200 mm deep. The average cubic compressive strength of three concrete testing cubes with dimensions of 150 mm × 150 mm × 150 mm was $f_{cu} = 87$ MPa over a period of 28 days. Fig. 1 and Fig. 2 show the specimen, loading set-up, cross section and longitudinal section of the tested specimens. Further details of the experimental program can be found in the study conducted by Yang [4].

2.1 NUMERICAL SIMULATION OF SLABS

As the member is asymmetric, a full size model was developed. The geometry, properties of the constituent materials, static loading, and boundary conditions in the developed FE models are similar to the tested specimens. Fig. 1(a) and (b) show experimental slab and typical FEA model that formed a basis of the present study. All slabs were monotonically loaded in flexure under a simply supported condition.

2.2 ELEMENTS TYPES DESCRIPTION

Fig. 1(b) shows a typical FEA model for the beams with prestressed steel tendons. For a typical FEA model of the steel prestressed ballastless track slabs, the concrete was modeled using a three dimensional composite solid element (SOLID 65). The size of the concrete element used for the beams studied here was from 30 to 50 mm.
mm, which was adequate element dimensions, given that the recommended size of a concrete element is one to three times aggregate size. The eight node element includes three degrees of freedom per node and is capable of representing concrete crushing and cracking in compression and tension, respectively, depending upon the level of applied stress as described in the previous section.

The steel tendons, SFCB and BFRP were modeled with a three-dimensional spar element (LINK 8) having two nodes per element. The spar element has three degrees of freedom per node and there is no discrepancy in terms of the degrees of freedom when compared with those of the concrete element. The effect of prestressing was given to the strain vector of the steel element, namely, the constitutive relationship of the steel tendon is \( \sigma = [K] \times [\varepsilon_m + \varepsilon_p] \), where, \( \sigma = \) stress vector, \( [K] = \) stiffness matrix, \( \varepsilon_m = \) mechanical strain vector, and \( \varepsilon_p = \) initial prestressing strain vector. Perfect bond was modeled for the constituent materials.

### 2.3 MATERIAL PROPERTIES

The behaviour of concrete was modeled using the William and Warnke approach as shown in Fig 3. The failure criterion of concrete subjected to multiaxial stresses was defined in the form of Eq. 1.

\[
\frac{F}{f_c} - S_{\text{fail}} \geq 0
\]

Where, \( F = \) function of the principal stress state in orthogonal directions and \( S_{\text{fail}} = \) failure surface function consisting of five parameters. Once the stress state satisfies Eq. 1, the concrete element fails.

![Figure 3 Conceptual view of the constitutive models for concrete in the FEA: a: three-dimensional failure surface in principal stress domain; b: concrete strength of cracked condition in tension](image)

The nonlinear material behavior of the steel reinforcement was simulated as elastic fully plastic based on the von-misses yield criteria and the BFRP was modeled as elastic brittle material until failure while the SFCB was modeled as bi-linear elastic material until failure. All adopted values of the reinforcements properties are shown in Table.1

<table>
<thead>
<tr>
<th>Bar type</th>
<th>( d ) (mm)</th>
<th>( E_1 ) (GPa)</th>
<th>( E_2 ) (GPa)</th>
<th>( f_y ) (MPa)</th>
<th>( f_u ) (MPa)</th>
<th>Elongation rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BFRP</td>
<td>16</td>
<td>44</td>
<td>——</td>
<td>1279</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>S6B27</td>
<td>11</td>
<td>100</td>
<td>40</td>
<td>256</td>
<td>872</td>
<td>2.7</td>
</tr>
<tr>
<td>Steel</td>
<td>8</td>
<td>195</td>
<td>——</td>
<td>555</td>
<td>644</td>
<td>16.2</td>
</tr>
<tr>
<td>Steel</td>
<td>20</td>
<td>195</td>
<td>——</td>
<td>554</td>
<td>713</td>
<td>25.3</td>
</tr>
<tr>
<td>Steel</td>
<td>16</td>
<td>200</td>
<td>——</td>
<td>550</td>
<td>609</td>
<td>24.6</td>
</tr>
<tr>
<td>Steel</td>
<td>10</td>
<td>200</td>
<td>——</td>
<td>1449</td>
<td>1725</td>
<td>9.5</td>
</tr>
<tr>
<td>Steel</td>
<td>6</td>
<td>205</td>
<td>——</td>
<td>482</td>
<td>685</td>
<td>20.0</td>
</tr>
</tbody>
</table>

### 3. VALIDATION OF THE PREDICTIVE MODELS
The FEA results are compared with the experimental results in Fig.4 in terms of the load-deflection and load-strain response at mid-span. The predicted cracking loads are slightly lower than those of the experimental slabs for the FEA model. This observation may be due to the fact that the models take the empirical modulus of rupture $f_r = 0.6 \sqrt{f_c}$ to represent the cracking load. For the ultimate loads, the FEA and the iterative models exhibit an average error of 3%, respectively, when compared to the experiments. For the ultimate strain, an average error of 10% is observed. These observations may be explained by the fact that the predictive slabs had an ideal concrete-and-tendon interaction. Overall, the predictive modeling approaches agree well with the experimental data.

Figure 4 Comparison of the modelling approach (a) load-deflection curve; (b) load-strains curve

![Figure 4](image1.png)

(a) load-deflection curve

(b) load-strains curve

Figure 5 Load-deflection curve of the FEA model in different prestressed levels

![Figure 5](image2.png)

(a) S-GX-SFCB

(b) S-GX-RC

(c) S-GX-BFRP

Figure 5 Load-deflection curve of the FEA model in different prestressed levels
Fig. 5 shows the behavior of FEA beams, depending upon the level of prestress. The cracking load is significantly influenced by the prestress levels, whereas the ultimate load is essentially the same irrespective of the prestress effect. The failure mode of the beams is governed by the concrete rupture in most cases, when the concrete strain reaches 0.001. When the prestress level decreases, an offset in the member response is observed; however, such a trend is not found in the nonprestressed beam 0% prestressing. For SFCB and BFRP slabs, bilinear and linear responses are found in Fig. 5 (a) and (c), respectively.

4. CODE PREDICTION

A comparison of the effective moment of inertia for prestressed slabs is made in Fig. 5, including the FEA, ACI-440 provisions (Eq. 2), and Bischoff’s equation (Eq. 3). ACI Committee 440.4R-04 suggests Eq. 2 to predict the effective moment of inertia for FRP-reinforced or FRP-prestressed members. It should be noted that the ACI 440.4R-04 equation for FRP-prestressed concrete has been originally developed for nonprestressed members, given that ACI440.4R-04 has adopted the provision of ACI440.1R-03 American Concrete Institute ACI 2003 that is for FRP reinforced concrete, and Bischoff’s equation was rationally derived from the load deflection response of an FRP-reinforced concrete beam, rather than empirical curve fitting.

\[
I_e = (M_{cr}/M_a)^\beta I_g + [1-(M_{cr}/M_a)] I_{cr} \leq I_g \quad (2)
\]

\[
I_e = I_{cr}/(1-\eta \gamma (M_{cr}/M_a)^2) \leq I_g \quad (3)
\]

Where \( \beta = 0.5(E_p/E_s) \), \( \eta = 1 - I_{cr}/I_g \), and for Series S-KZ, \( \gamma = ((3a/L) - 4(\zeta a/L))/((3a/L) - 4(\zeta a/L)) \), where \( \zeta = 4(M_{cr}/M_a) - 3 \). For Series S-GX, \( \gamma = 3 - 2(M_{cr}/M_a) \), as shown in Table 4, \( M_{cr} \) is the cracking moment (considering the pre stress on the cross section), \( I_g (= bh^3/12) \) is the gross moment of inertia for a rectangular section about the horizontal censorial axis of the cross-section, \( M_a \) is the service moment in the member, and \( I_{cr} \) is the moment of inertia of the transformed crack section in Eq. 4, which is derived from an elastic cracked-section analysis as follows:

\[
I_{cr} = \frac{1}{3} bx_{cr}^3 + (n_s A_s) (h_0 - x_{cr})^2 + (n_s' A_s) (h_1 - x_{cr})^2 + (n_s A_s') (h_2 - x_{cr})^2 \quad (4)
\]

Where, \( x_{cr} \) is the compression zone of concrete, \( n_s ( = E_{sf}/E_c) \) is the elastic modulus ratio between the FRP reinforcement and concrete, \( n_s ( = E_s/E_c) \) is the elastic modulus ratio between the steel reinforcement and concrete, \( d \) is the distance from the extreme compression fiber to the centroid of the tension reinforcing zone, and \( h_0, h_1, \) and \( h_2 \) are the distance from the extreme compression fiber to the centroid of the tension reinforcing zone in the upper reinforcement, under reinforcement, and prestressed bars in the cross section, respectively.
Comparisons of the effective moment of inertia with the FEA results for slabs are shown in Fig. 6, whenever the experimental responses are not available. For all slabs with $I_g/I_{cr} = 14$, the FEA response is close to the predictions with prestress levels greater than 40% ultimate strength, whereas the response approached none of them when the prestress level decreased. In addition, when the prestress levels greater than 80%
ultimate strength, Bischoff’s method underestimates the deflection. Fig. 6 (g) shows ACI440.4R-04 predicts lower deflection comparing with Bischoff’s method in any load level.

5. CONCLUSIONS

Based on the experimental results of ordinary steel, SFCB and BFRP reinforced ballastless track slabs; an elaborate finite element model was proposed for simulating the nonlinear behavior of ballastless track slabs using the Ansys 13.0. The procedures for element selection and material modeling were presented in detail using an elastic-plastic constitutive model. The numerical model considering both the material and the geometric nonlinearities could fully reflect the complex characteristics such as load-deflection, the strain changes, etc. As the finite-element results are in good agreement with the test results, the proposed finite-element model provides a tool for non-linear analysis of ballastless track slabs. In addition, comparisons of the effective moment of inertia with the FEA results for slabs were presented. With prestress levels between 40% ultimate strength to 60% ultimate strength, the FEA model get close to code predications. However, prestress levels greater than 80% ultimate strength, Bischoff’s method underestimated the deflection. And ACI440.4R-04 predicted lower deflection comparing with Bischoff’s method in any load level with $I_e/I_{cr} = 14$.

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