ULTIMATE LOAD MODELING OF RC FLATS ELEMENTS STRENGTHENED BY CFRP

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ABSTRACT

This paper presents the experimental investigations and a specific failure criterion for evaluating the ultimate punching load of concrete slabs strengthened by composite materials. First, the flexural mode failure is calculated by the yield lines method to ensure that it was the punching phenomenon that caused the break. The analytical model for punching failure of reinforced concrete slabs developed by Menétrey is used. For RC slabs reinforced by composite materials, the punching load is calculated by the Rochdi enhancement model. Finally, the experimental values are compared to the analytical model values to validate the method of calculation.

KEYWORDS

CFRP, reinforcement, RC Slab, failure criteria

INTRODUCTION

The increasing number of building and civil engineering work pathologies mean that reinforcement solutions are required. While much research on RC structures reinforced by external bonded CFRP has been carried out in recent years, these studies mainly concern beams and columns; little research has been done on reinforced concrete slabs. The two-way span and the thinness of these structures result in complex behaviours. When reinforced, the bending stiffness and the bearing capacity of the slab increase while some new failure modes appear. The punching phenomenon leads to premature failure of structures.

The punching shear criterion was developed by Menétrey [P. Menétrey, 2002] and modified by Rochdi et al [E.H. Rochdi et al, 2004] in the case of CFRP plates externally bonded on the whole slab surface. We propose to improve this failure criterion so as to consider the reinforcement discontinuity that occurs when bonding with CFRP strips as well as the three cracking angles and the presence of steel rebars on the top surface. Not all of these parameters were taken into account either on the Menétrey method or on the Rochdi criteria.

In the original model the authors show that the punching load \( F_{pun} \) of the slab is of course balanced with the vertical component of the tensile force supported by the concrete at the moment of punching \( F_{ct} \) and with the dowel effect induced by the steel bars \( F_{dows} \). If a compressive steel reinforcement is added, the shear crack propagation changes and the previous vertical forces may be subdivided in many terms for concrete \( F_{ct1} \) and \( F_{ct2} \) as well as \( F_{dows1} \) and \( F_{dows2} \) for steel reinforcement. Angles \( \alpha_1 \) and \( \alpha_2 \) are experimentally measured. (Figure 1)

![Figure 1. Punching shear capacity of reinforced concrete slab](image)
PUNCHING SHEAR FAILURE CRITERION

Tensile Strength in Concrete: Fct1 and Fct2

Fct1 calculation:
The ultimate tensile strength of concrete is low. It is calculated by integrating the vertical component along the shear cracks between the distance \( l_{1s} \) and \( l_{2s} \). The \( l_{1s} \) and \( l_{2s} \) length are calculated as follows:

\[
l_{1s} = \frac{C_1}{2} \quad \text{and} \quad l_{2s} = \frac{C_1}{2} + \frac{d_1}{\tan \alpha_1}
\]

(1)

with: \( \alpha_1 \): punching cracks angle (experimental value approved by EC2, \( \alpha_1 = 34^\circ \)), \( C_1 \): width of the loading surface [mm], \( d_1 \): effective depth of the slab [mm]

Determination of the leaning cracks length \( s_1 \) is done by:

\[
s_1 = \sqrt{(l_{2s} - l_{1s})^2 + (d_1)^2}
\]

(2)

Tensile shear strengths are assumed to be uniform along shear punching cracks. The vertical component force \( F_{ct1} \) is given by:

\[
F_{ct1} = (\pi \cdot (l_{1s} + l_{2s})) \cdot s_1 \cdot \sigma_{v1}
\]

(3)

with: \( \sigma_{v1} \): tensile concrete stress [MPa].

The tensile concrete force is influenced both by the mechanical properties of the concrete and the ratio of steel. This is taken into account through the factor access determined numerically by Menétrey. Simulation results show that the parameter \( \xi_1 \) follows a parabolic variation which depends on the rebar ratio \( \rho_1 \).

\[
\xi_1 = -0.1 \cdot \rho_1^2 + 0.46 \cdot \rho_1 + 0.35
\]

(4)

The thickness of the slab is also taken into account by the parameter \( \mu_1 \). This parameter is given by the following relation:

\[
\mu_1 = 1.6 \cdot \left( 1 + \frac{d_1}{d_a} \right)^{-1/2}
\]

(5)

with: \( d_1 \): effective depth of the slab [mm], \( d_a \): maximal concrete aggregate diameter [mm].

The loading surface is also taken into account in the calculation by the factor \( \eta_1 \). This parameter is obtained by numerical simulation taking into account several ratios \( C/h \).

\[
\eta_1 = \begin{cases} 0.1 \cdot \left( \frac{C}{h} \right)^2 + 0.5 \cdot \left( \frac{C}{h} \right) + 1.25 & \text{if } 0 < \left( \frac{C}{h} \right) < 2.5 \\ 0.625 & \text{if } \left( \frac{C}{h} \right) \geq 2.5 \end{cases}
\]

(6)

The tensile strength in concrete \( F_{ct1} \) is given by following relation:

\[
F_{ct1} = \left( \pi \cdot (l_{1s} + l_{2s}) \right) \cdot s_1 \cdot f_{ctm}^{2/3} \cdot \xi_1 \cdot \mu_1 \cdot \eta_1
\]

(7)

Fct2 calculation:
The force acting on the concrete between two layers of steel rebars \( F_{ct2} \) is calculated in the same way as the one of the top of the slab \( F_{ct1} \). Calculation of the \( l_{1i} \) and \( l_{2i} \) length gives:

\[
l_{1i} = \frac{C_i}{2} + \frac{d_i}{\tan \alpha_1} \quad \text{and} \quad l_{2i} = \frac{C_i}{2} + \frac{d_i}{\tan \alpha_2} + \frac{d_2 - d_1}{\tan \alpha_2}
\]

(8)

with: \( \alpha_1 \): punching cracks angle (experimental value approved by EC2, \( \alpha_1 = 34^\circ \)), \( \alpha_2 \): punching cracks angle between two beds (\( \alpha_2 = 20^\circ \)), \( C_i \): width of the loading surface [mm], \( d_i \): effective depth of the top bars [mm], \( d_2 \): effective depth of the lower bars [mm],

Calculation of the leaning cracks length: \( s_2 \)

\[
s_2 = \sqrt{(l_{2i} - l_{1i})^2 + (d_2)^2}
\]

(9)

The vertical component force \( F_{ct2} \)

\[
F_{ct2} = \left( \pi \cdot (l_{1i} + l_{2i}) \right) \cdot s_2 \cdot \sigma_{v2}
\]

(10)

Parameter \( \xi_2 \)

\[
\xi_2 = -0.1 \cdot \rho_2^2 + 0.46 \cdot \rho_2 + 0.35
\]

(11)

Parameter \( \mu_2 \)
\[ \mu_i = 1.6 \left( \frac{d_i}{d_{cr}} \right)^{0.2} \]  

(12)

Factor \( \eta_2 \):

\[ \eta_2 = \begin{cases} 0.1 \left( \frac{C}{h} \right)^2 - 0.5 \left( \frac{C}{h} \right) + 1.25 & 0 < \left( \frac{C}{h} \right) < 2.5 \\ 0.625 & \frac{C}{h} \geq 2.5 \end{cases} \]

(13)

The tensile strength in concrete \( F_{ct2} \) is given by following relation:

\[ F_{ct2} = (\pi \cdot (l_i + l_2)) \cdot s_2 \cdot f_{com}^{0.5} \cdot \eta_2 \cdot \mu_2 \cdot \eta_2 \]

(14)

**Steel Reinforcement Force: Fdows1 and Fdows2**

The steel rebars are usually designed to sustain normal stress. Steel trusses with different diameter and spacing are used. But these rebars can also sustain vertical stress when shear cracks appears. This is the dowel action of bars. This effect interprets the amount of shear force that can be transferred by reinforcing bars crossing the punching crack.

For the calculation of the rebars’ contribution, Menétrey considers the development proposed by CEB-FIP.

\[ F_{dows,i} = \frac{1}{2} \sum_{bars} \phi_{s,i} \sqrt{f_{ck} \cdot f_{yk} \cdot (1 - \zeta_{s,i}^2)} \cdot \sin \alpha_i \]

(15)

with: \( \Phi_{s,i} \): diameter of bar \( i \) [mm], \( f_{ck} \): concrete compressive strength [MPa], \( f_{yk} \): steel yield stress [MPa], \( \zeta_{s,i} \): work ratio of bar \( i \), \( \alpha_i \): punching failure angle, \( i \): number of rebars layer

A parabolic interaction is assumed between the axial force and the dowel force in the reinforcing bar which is expressed by the term \( (1-\zeta_{s,i}^2) \) where \( \zeta_{s,i} = \sigma_s / f_{yk} \) and \( \sigma_s \) is the axial tensile stress in the reinforcing bars which have a yield strength \( f_{yk} \). The axial tensile stress \( \sigma_s \) is obtained by the projection of vertical forces in the rebars onto inclined cracks divided by the total area of rebars crossing the punching crack.

\[ \zeta_s = \frac{\sigma_s}{f_{yk}} \quad \text{with} \quad \sigma_s = \frac{F_{yi}}{\tan \alpha} \sum_{bars} A_i \]

(16)

The rebars’ contribution is reduced by \( \sin \alpha \) to take account of the angle between cracks and rebars. This model can be applied to any kind of reinforcement (pre-stressing, punching shear reinforcement etc.). In our case we obtain:

For top rebars:

\[ F_{dows1} = \frac{1}{2} \sum_{bars} \phi_{s1} \sqrt{f_{ck} \cdot f_{yk} \cdot (1 - \zeta_{s1}^2)} \cdot \sin \alpha_1 \]

(17)

\[ \zeta_{s1} = \frac{\sigma_{s1}}{f_{yk}} \quad \text{and} \quad \sigma_{s1} = \frac{F_{ct1}}{\tan \alpha_1} \sum_{bars} A_i \]

(18)

For lower rebars:

\[ F_{dows2} = \frac{1}{2} \sum_{bars} \phi_{s2} \sqrt{f_{ck} \cdot f_{yk} \cdot (1 - \zeta_{s2}^2)} \cdot \sin \alpha_2 \]

(19)

\[ \zeta_{s2} = \frac{\sigma_{s2}}{f_{yk}} \quad \text{and} \quad \sigma_{s2} = \frac{F_{ct2}}{\tan \alpha_2} \sum_{bars} A_i \]

(20)

To take into account the fact that the rebars do not pass through the crack at an exact right angle, the coefficient \( \frac{1}{2} \) is introduced in expression of \( F_{dows1} \) and \( F_{dows2} \).

**Punching Load Slab Capacity: Fpun**

Finally in the case of slab with two layers of steel, the punching capacity of the slab can be expressed by the sum of concrete and steel tensile force vertical components.

\[ F_{pun} = F_{ct1} + F_{ct2} + F_{dows1} + F_{dows2} \]

(21)

**PUNCHING SHEAR FAILURE CRITERION WITH FRP**

As mentioned before, Menétrey’s method takes into consideration the sum of the vertical forces of each component. It is possible to adapt this method for the case of reinforced slabs and to consider the composite material effects. Therefore the punching load capacity can be obtained by the following relation:
where $F_{\text{dowf}}$ corresponds to the effects of CFRP.

Expression of $F_{\text{dowf}}$ has been developed by Rochdi et al. [E.H. Rochdi et al, 2004] in the case of continuously reinforced slabs, i.e. where the composite laminate is bonded on the whole lower surface of the slabs. This solution is not optimal in term of material cost. Another solution is to use composite strips. In this latter case, we have to adapt the Rochdi model. In both cases, contributions of concrete and steel components are calculated in the same way as for non-reinforced slabs. The CFRP contribution follows the same kind of expressions as for the steel rebars.

**Continuously Reinforced Slabs**

Rochdi et al. approximated the stress in CFRP while considering a bi-directional (2D) composite. The first step of this approach consists of assessing the critical perimeter $l_c$ corresponding to the punching load.

$$l_c = 2 \cdot (C_1 + C_2) + \frac{2 \cdot \pi \cdot h}{\tan \alpha_s} \tag{23}$$

As the reinforcement ratio does not in fact act on the critical perimeter, the angle $\alpha_s$ can be considered as equal to the sum of punching crack angles.

$$\alpha_s = \frac{d_1 \cdot \alpha_1 + (d_2 - d_1) \cdot \alpha_2 + (h - d_2) \cdot \alpha_3}{h} \tag{24}$$

Unlike steel, the mechanical behaviour of CFRP is anisotropic. The change of stress direction has to be taken into account during the calculation. Several axes of stress should be considered: 0°, 90° and ±θ. (Figure 2).

![Figure 2. Critical perimeter of slab reinforced entirely by composite material](image)

To simplify the calculation, the circular part of the critical perimeter is divided into three straight lines with identical length $l_{\text{cir}}$, characterized by average orientations of 15°, 45° and 75° respectively. $F_{\text{dowf}}$ is then equal to the sum of the tensile forces acting on the CFRP along the critical lengths $l_{\text{cir}}, l_{\text{C1}}$ and $l_{\text{C2}}$.

$$F_{\text{dowf,0°}} = e_f \cdot \sqrt{f_{\text{ck}} \cdot f_{f(0°)} \cdot (1 - \frac{2}{\pi}) \cdot \sin \alpha_s} \cdot C_1 \tag{25}$$

$$F_{\text{dowf,90°}} = e_f \cdot \sqrt{f_{\text{ck}} \cdot f_{f(90°)} \cdot (1 - \frac{2}{\pi}) \cdot \sin \alpha_s} \cdot C_2 \tag{26}$$

$$F_{\text{dowf,θ°}} = e_f \cdot \sqrt{f_{\text{ck}} \cdot f_{f(θ°)} \cdot (1 - \frac{2}{\pi}) \cdot \sin \alpha_s} \cdot l_{\text{cir}} \tag{27}$$

A quadratic interaction between axial and vertical CFRP forces is also assumed to be the same as in previous expressions. We define the following composite work ratios $\zeta_{(0°)}$, $\zeta_{(90°)}$ and $\zeta_{(θ°)}$.

$$\zeta_{f(θ°)} = \frac{\sigma_{f(θ°)}}{f_{f(θ°)}} \quad \zeta_{f(90°)} = \frac{\sigma_{f(90°)}}{f_{f(90°)}} \quad \zeta_{f(θ°)} = \frac{\sigma_{f(θ°)}}{f_{f(θ°)}} \tag{28}$$

with : $\sigma_{f(θ°)}$ : composite stress in the direction 0°; $\sigma_{f(90°)}$ : composite stress in the direction 90°; $f_{f(θ°)}$ : composite stress in the direction 0°; $f_{f(90°)}$ : bi-axial strength in the direction 0°; $f_{f(θ°)}$ : bi-axial strength in the direction 90°; $f_{f(θ°)}$ : bi-axial strength in the direction θ°.

$$\sigma_{f(θ°)} = \frac{F_{\text{ck}}}{e_f \cdot 2 \cdot (C_1)} \quad \sigma_{f(90°)} = \frac{F_{\text{ck}}}{e_f \cdot 2 \cdot (C_2)} \quad \sigma_{f(θ°)} = \frac{F_{\text{ck}}}{e_f \cdot 2 \cdot (l_{\text{cir}})} \tag{29}$$

Thus we write in the general case:

$$F_{\text{dowf}} = 2 \times F_{\text{dowf,0°}} + 2 \times F_{\text{dowf,90°}} \sum_{θ} F_{\text{dowf,θ°}} \tag{30}$$

and for a bi-directional carbon sheet : $F_{\text{dowf,90°}} = F_{\text{dowf,0°}} F_{\text{dowf,75°}} = F_{\text{dowf,15°}}$
Hence:

\[
F_{dowf} = 4 \times F_{dowf,0^\circ} + 4 \times F_{dowf,45^\circ} + 8 \times F_{dowf,15^\circ}
\]  

(31)

**Slab Reinforced by a Crossed CFRP Strip**

Rochdi *et al.* method can be extended to this case. It is necessary to evaluate the critical perimeter. It is calculated as the sum of each composite width located on the failure perimeter. The area of stressed composite may be calculated with the thickness of the composite.

![Critical perimeter of slab reinforced by composite strip](image)

Figure 3. Critical perimeter of slab reinforced by composite strip

Critical perimeter determination:

\[
l_c = \sum l_{ci}
\]  

with: \( l_{ci} = l_{\gamma} \), \( l_{\beta} \) ...

\[
\left. \begin{array}{l}
\xi_{f(0)} = \frac{\sigma_{f(0)}}{f_{b(0)}} \\
\sigma_{f(0)} = \frac{F_{\alpha}/\tan \alpha}{e_{f} \cdot l_{\alpha}}
\end{array} \right\}
\]  

(32)

\[
F_{dowf} = \sum_{f} e_{f} \cdot \sqrt{f_{c,k} \cdot f_{b(\gamma)} \cdot (1 - \xi_{(\gamma)}^{2}) \cdot \sin \alpha_{\gamma} \cdot l_{c,\gamma}}
\]  

(33)

In this case:

\[
\gamma = \frac{\theta_{G1} + \theta_{G2} + \theta_{G2}}{2} \quad \text{and} \quad \beta = \frac{\theta_{F1} + \theta_{F2} + \theta_{F2}}{2}
\]  

(34)

\[
l_{c,\gamma} = \left( \theta_{G1} - \theta_{G1} \right) \frac{2 \cdot \pi \cdot dF}{360} \quad \text{and} \quad l_{c,\beta} = \left( \theta_{F1} - \theta_{F1} \right) \frac{2 \cdot \pi \cdot dG}{360}
\]  

(35)

\[
\xi_{f(\gamma)} = \frac{\sigma_{f(\gamma)}}{f_{b(\gamma)}} \quad \text{and} \quad \sigma_{f(\gamma)} = \frac{F_{\alpha}/\tan \alpha}{e_{f} \cdot l_{c,\gamma}}
\]  

(36)

\[
F_{dowf,\gamma} = e_{f} \cdot \sqrt{f_{c,k} \cdot f_{b(\gamma)} \cdot (1 - \xi_{(\gamma)}^{2}) \cdot \sin \alpha_{\gamma} \cdot l_{c,\gamma}}
\]  

(37)

\[
\xi_{f(\beta)} = \frac{\sigma_{f(\beta)}}{f_{b(\beta)}} \quad \text{and} \quad \sigma_{f(\beta)} = \frac{F_{\alpha}/\tan \alpha}{e_{f} \cdot l_{c,\beta}}
\]  

(38)

\[
F_{dowf,\beta} = e_{f} \cdot \sqrt{f_{c,k} \cdot f_{b(\beta)} \cdot (1 - \xi_{(\beta)}^{2}) \cdot \sin \alpha_{\beta} \cdot l_{c,\beta}}
\]  

(39)

CFRP mechanical properties are calculated in function of the angle between composite main axis and perpendicular to the cracks. For each strip this value is calculated knowing the evolution of ultimate composite strength in the loading direction.

We undertake these values in the “Hill-Tsai criteria”:

\[
\sigma_{\alpha}^{2} \left\{ \frac{c^{4}}{\sigma_{i}^{2}} + \frac{s^{4}}{\sigma_{i}^{2}} - \frac{c^{2} \cdot s^{2}}{\sigma_{i}^{2}} + \frac{c^{2} \cdot s^{2}}{r_{a}^{2}} \right\} \leq 1 \quad \text{and} \quad \sigma_{s} = \frac{1}{\sqrt{\frac{c^{4}}{\sigma_{i}^{2}} + \frac{s^{4}}{\sigma_{i}^{2}} + c^{2} \cdot s^{2} \cdot \frac{1}{r_{a}^{2}} - \frac{1}{\sigma_{i}^{2}}}}
\]  

(40)
RC SLAB APPLICATION STRENGTHENED BY CFRP

To confirm the global Menétry method, the theoretical values are compared with experimental one thanks to an experimental investigation done by Michel and all. [L. Michel et al, 2007]

The geometrical data for the model are sum up in the Table 1.

<table>
<thead>
<tr>
<th>appointment</th>
<th>C1</th>
<th>C2</th>
<th>d1</th>
<th>d2</th>
<th>da</th>
<th>h</th>
<th>α1</th>
<th>α2</th>
<th>αs</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST65 C</td>
<td>100</td>
<td>100</td>
<td>35</td>
<td>70.5</td>
<td>20</td>
<td>100</td>
<td>30</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>RAF C</td>
<td>35</td>
<td>35</td>
<td>42.75</td>
<td>20</td>
<td>70</td>
<td>35</td>
<td>20</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

The length unit is mm and angle unit is degree.

Table 1: Geometrical data

The mechanical properties are resumed in the Table 2.

<table>
<thead>
<tr>
<th>Type of slab</th>
<th>$f_{ck}$ [MPa]</th>
<th>$f_{cm}$ [MPa]</th>
<th>$f_{ck}$ [MPa]</th>
<th>$\Phi_{sup}$ [mm]</th>
<th>Sheared reinforcement number</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST65 C</td>
<td>34</td>
<td>3.3</td>
<td>550</td>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>RAF C</td>
<td>34</td>
<td>3.3</td>
<td>550</td>
<td>4.5</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 2: Mechanical properties of materials

After the model application the punching shear values are sum up in the Table 3:

<table>
<thead>
<tr>
<th>Type of slabs</th>
<th>$F_{pun,th}$</th>
<th>$F_{pun,exp}$</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0</td>
<td>143</td>
<td>134.7</td>
<td>+6.1</td>
</tr>
<tr>
<td>R1</td>
<td>145.6</td>
<td>139.5</td>
<td>+4.4</td>
</tr>
<tr>
<td>R3</td>
<td>215.6</td>
<td>154.3</td>
<td>+39.8</td>
</tr>
<tr>
<td>R5</td>
<td>284.6</td>
<td>202.3</td>
<td>+40.6</td>
</tr>
<tr>
<td>R1 pultruded</td>
<td>171.4</td>
<td>204.7</td>
<td>-16.3</td>
</tr>
<tr>
<td>R0</td>
<td>21.2</td>
<td>25.4</td>
<td>-16.5</td>
</tr>
<tr>
<td>R1</td>
<td>55.5</td>
<td>52.8</td>
<td>+5.1</td>
</tr>
<tr>
<td>R3</td>
<td>124.3</td>
<td>111.9</td>
<td>+11</td>
</tr>
<tr>
<td>R5</td>
<td>193.1</td>
<td>117.6</td>
<td>+64.3</td>
</tr>
<tr>
<td>R1 pultruded</td>
<td>74.7</td>
<td>67.1</td>
<td>+11.3</td>
</tr>
</tbody>
</table>

Table 3: Results of the complete Menétry model (The load unit is kN.)

The obvious differences between the experimental and the theoretical results of slabs reinforced by composite materials is due to the assumptions in the Menétry model, which does not consider the tensile break in composite fibers, the shear break in the glue joint nor the tensile or shear failure in the concrete.

CONCLUSION

The punching shear stress calculation based on the Menétry approach is developed and applied to the case of RC slabs externally reinforced by composite materials. The analyzed slabs are continuously reinforced by a cross-laid CFRP strip. The proposed model for punching failure load is more accurate than previous models (the Rochdi method). In the case of RC slabs, the difference between experimental and analytical values is about 35% for the Rochdi model, and about 5% for the proposed method. However, with RC slabs externally reinforced by CFRP, the difference is greater (about 15%). In order to reduce the difference between experimental and theoretical results, all types of failure (CFRP/concrete debonding, peeling-off failure...) will in future be considered in the calculation.

REFERENCES


